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Mispricing in the Black-Scholes model: an exploratory analysis

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CHAPTER 1. INTRODUCTION

The pace of innovation in financial markets has increased in recent years. Call option trading in sixteen individual common stocks was authorized only twenty years ago on the Chicago Board Options Exchange (CBOE). Today put options as well as call options on about 360 individual stocks are traded on five exchanges-the American Stock Exchange (AMEX), the Philadelphia Stock Exchange, the Pacific Coast Exchange, the Midwest Stock Exchange, as well as on the CBOE-and the volume of the stock option trading measured by the value of the underlying contracts rivals that of the New York Stock Exchange (NYSE). This period has also seen the introduction of futures contracts on financial instruments, such as government securities and other debt instruments, currencies, and stock indexes. The most recent innovation has been the creation of options on instruments other than individual common stocks: bonds, stock indexes, currencies, and commodities.

A tremendous amount of research work has also been developed since the discovery of the option pricing model by Black and Scholes in 1973. The empirical research on options has focused on either testing the Black-Scholes model price and its underlying assumptions or developing an alternative call option pricing model. The study of options is useful for understanding not only the behavior of option prices but also of stock prices.

The main part of this study is the investigation of the mispricing of the Black-Scholes model. However, in order to calculate the model price, the stock volatility needs to be estimated first. These estimated stock volatilities have been documented by many studies to be sensitive to the change in option positions, i.e., maturities and also striking prices. Thus, the study examines whether the stock volatilities implicit in option prices vary across maturities, or striking prices, or both. The results of these tests provide the foundation for investigating the mispricing of the Black-Scholes model.

Based upon the acceptance of the Black-Scholes model with flexible implied volatility across maturity month and degree of moneyness, a robust model is built by incorporating the linear effect of exercise opportunity and time to maturity directly into the Black-Scholes model. In addition, as documented by many studies that the mispricing of the model is serially correlated, the error term of the robust model is assumed to follow a first order autoregressive process. Furthermore, in order to investigate the systematic mispricing of the Black-Scholes model across option positions, all the factors affecting call option prices are specified additively across option positions. As the result, the model is so general that it can be used to not only investigate the systematic mispricing of the Black-Scholes model, but also to check the implication of the Black-Scholes assumption, examine the behavior of the at-the-money implied volatilities that are claimed to be stable over short maturities.

In this thesis, the basic principles of regular call option valuation are explained in Chapter 2. The valuation process begins by determining the per share value of a regular call option at expiration. Then, the lower and upper bounds of regular call option are determined. The last part of the Chapter shows how to compute the Black-Scholes model price by using simple hand calculations.

In Chapter 3, the famous Black-Scholes option pricing model is derived. The Black-Scholes option price is the solution to the general equilibrium pricing frame work. However, the Black-Scholes formula can only apply to European call options. Therefore, it can only be used for valuing American call options if early exercise is most unlikely. Fortunately, the Black-Scholes formula can be generalized to include the case of American call options on dividend-paying stocks. This extended model will be discussed later at the end of Chapter 2.

Most of the discussion in Chapter 4 concerns testing the validity of the Black-Scholes model for pricing of the call options. A survey of the literature reveals four major approaches for testing the model's validity. The first approach is based on creating neutral hedge positions and testing the behavior of the returns from the investment. The approach eliminates the problem of risk-adjustment for investments in options. Thus, the option market efficiency can be tested using this technique. The second approach is based on imputing the standard deviations from actual option prices by using a pricing model. The behavior of the implied standard deviation is then investigated to determine the validity of the assumed model. The third approach uses a direct comparison of the model prices to actual prices. The tests are intended to show whether model prices are unbiased estimators of the actual prices or whether there are consistent deviations between the model and market prices. The fourth approach for testing pricing models is by means of simulations of deviations from the basic assumptions of the models. According to this approach the sensitivity of the model prices to empirical deviations from the assumptions is tested.

The main body of the study is contained in Chapter 5. This chapter also includes the review of literature investigating the systematic mispricing of the Black-Scholes model. The development of the robust model begins with testing the implications of the Black-Scholes model assumption of constant implicit volatility across degree of moneyness and maturity months. Then, a robust model is developed based on the results of the test. The robust model also corrects the problem of specification error that has been faced with the convention analysis in the past by incorporating the linear effect of striking price and maturity, and also the first order moving average directly into the model. With these modifications, the systematic mispricing of the Black-Scholes model can be meaningfully investigated. The thesis also examines the behavior of the at-the-money implied volatility and its mispricing behavior. Furthermore, the study compares the results of the mispricing obtained from the robust model to the results of two conventional analyses. It turns out that the regression results of the conventional analysis may misinterpret the mispricing of the Black-Scholes model. For another conventional analysis that directly computes the means of mispricing, the study shows that the precision of the estimates obtained from this conventional analysis is lower than that of the robust model.

The robust model also offers a way to estimate the implied volatility that is stable across striking prices and maturities. As a result, the estimate may be more closely related to the variance rate of stock return than the at-the-money short maturity implied volatility. As many studies have used the at-the-money short maturity implied volatility to analyze, for example, the impact of stock split, stock dividend, dividend policy, and their announcement effects to the changes in the variance rate of stock return. The estimated implied volatility obtained from the robust model can be used instead of. Thus, the robust model is useful for further research in not only the option area but also in the stock area.

CHAPTER 2. VALUATION OF THE CALL OPTION IN GENERAL

2.1 Overview

An option contract gives the holder the right to buy (in the case of a call option) or to sell (in the case of a put option) a specified quantity of a security (called the underlying security) at a specified price (called the exercise price or striking price). Conversely, the writer of the option contract is obligated to sell (in the case of a call option) or to buy (in the case of a put option) that quantity of the security at the exercise price. The trading unit is typically one hundred shares of stock, although under certain circumstances it may be more than this amount.

The buyer of the option pays a premium to the seller to secure the rights of buying or selling the underlying security at the exercise price. This premium is the market price of the option. The holder of the option may choose to exercise the right to buy or sell at any time during the life of the option; if the investor does not choose to do so, the option expires and is worthless after the expiration date.

Listed options are standardized puts or calls traded on a national securities exchange or in the over-the-counter market via the NASDAQ system (NASDAQ is a computer-oriented, broad based indicator of activity in the unlisted securities market, updated every five minutes). The Option Clearing Corporation (OCC) is issuer and guarantor of exchange-traded and NASDAQ put and call options. It also serves as the clearing agency for options transactions. Other over-the-counter options are traded directly between buyer and seller, and have no secondary market and no standardization of striking prices and expiration dates.

In general, options are traded during the same hours in which the underlying securities are traded in their primary markets. Options trading continues for ten minutes after the close of each day's stock trading (4:10 P.M. Eastern time). On the business day prior to the expiration date, all options cease trading at 4:00 P.M. Eastern time.

In 1971, the Chicago Board of Trade (CBT), the major U.S. futures contracts exchange, began research and development of an exchange-traded (listed) option markets. Two years later, the Chicago Board Options Exchange (CBOE) opened with trading in call options on sixteen underlying stocks. The growth of the listed option market was so great that many new stocks were added to the list. Within three years, a trio of similar exchanges were launched, and within a decade options trading column exceeded that of the underlying stocks, on a shared-equivalent basis.

The ability to easily buy and sell stock options also spawned a variety of new uses for them, and sophisticated investors quickly became familiar with complex option strategies, such as, butterfly spreads and straddles, as a means to diversify and hedge a portfolio. The enormous appeal of options, particularly at a time when interest in securities was subdued, prompted Wall Street to go one step further and create kindred instruments that were unburdened by an underlying instrument, namely options on stock indexes and options on stock-index futures. These inventions enabled an investor to buy the equivalent of an entire basket of stocks on margin that was less than half that required to buy the equities themselves.

All options traded on the CBOE are written on stocks traded on the New York Stock Exchange (NYSE). Therefore, trading in the CBOE takes place during the hours when the NYSE is open for trading. In order to create and maintain a liquid, active market, the CBOE had to standardize the terms of the options. The standardization concerns maturity dates and striking prices. The CBOE declared 4 months to be expiration months (January, April, July, and October) and the expiration time to be 11:59 P.M. Eastern time on the Saturday immediately following the third Friday of the expiration month. The maximum duration of a CBOE option is, thus, 9 months, and therefore, at any given time, there are options available for at least 3 expiration months. The expiration time is not the same as the earlier exercise cutoff time. In order to exercise an option, a customer must so instruct his broker prior to the exercise cutoff time. For brokers who are members of the National Association of Securities Dealers (NASD) or of an exchange, the exercise cutoff time is 5:30 P.M. Eastern time, on the business day immediately preceding the expiration date, the latest time is 8:00 P.M. Eastern time. The striking prices are always in multiples of \$5 (or \$10 for the stock above \$100). On the first day of an option, the striking price will be that multiple of \$5 which is the closet to the market price of the underlying stock at that time.

In June, 1976 the American Stock Exchange also began trading listed options. The success and growth of both options market places brought about the opening of three other exchanges to such trading: the Pacific Exchange, the Philadelphia Stock Exchange and, in 1984, the New York Stock Exchange. In addition, trading in NASDAQ options commenced in 1985.

2.2 Call Option Valuation

In this section, the basic principles of regular call option valuation will be explained. The Black-Scholes formula is presented in the following section. In general, the per share value of a regular call option is presented as a function of the current share price of its underlying stock. The reason why people concentrate on the per share value of an option is that option prices are always quoted on a per share basis, even though , an option contract usually covers 100 shares. In addition to the current share price, the per share value of a regular call option also depends on a number of other objective factors-namely its exercise price, its time to expiration, and the risk-free rate of interest as well as on investor's subjective expectations with respect to the future behavior of the stock price.

The exercise price is the contractual share price at which a fixed amount of shares (usually 100) of the underlying stock can be purchased by the option holder when exercising the option. Usually this exercise is a multiple of $5 or $10, when established, option contracts expire about nine months later. The risk-free rate of interest is commonly considered to be the yield on treasury bills with roughly the same expiration date as the option.

For the time being, the call option value per share (V_o) will be expressed as a function, f, of only three determinants: the current share price (S), the exercise price (K), and the remaining time (T), as shown in the Equation 2.1:

$$V_o \quad = \quad f(S,K,T) \qquad (2.1)$$

Both S and T are the only two arguments of function (2.1) that vary over the life of the option, whereas K is usually an invariable option feature that is fixed at the time of issue. Firstly, the analysis of the above function focuses on the way in which the option value depends on the current stock price level, assuming a given time to maturity. Next, the changes in the value of an option when its remaining life shrinks as time goes on will be studied.

From the outset, it is useful to point out that, other things being equal, the option value per share would be *halved.* This can be shown as follows. Suppose that an investors has a number of options to buy 100 shares of a particular common stocks at an exercise price K before a certain expiration date, and suppose there is a 2-for-1 stock split[1]. All other things

[1] **Firm's policy to double the number of shares outstanding. Thus, the earnings, dividends per share, and the stock price will be halved.**

being equal, such a stock split would cut the share price in half. Unless there are contractual provisions to adjust the terms for exercising the options in such a case, this share price reduction would decrease the position of the investor, as he could probably no longer exercise his options at a price that is attractive relative to the new share price. The value of the options could, however, be immunized against such a stock split by a contractual provision for (1) cutting the exercise in half and (2) doubling the number of shares the investors is entitled to buy. As a result, the investor would be able to acquire the same fraction of all outstanding stock for the same total dollar investment as before the stock split. In addition, since the value of the company has not been changed by the stock split, the acquired shares of common stock still have the same value.

To illustrate this point, assume that the stock is priced at $50 a share, and that the exercise price of the option is $45. If the total number of outstanding shares is 1,000,000, and if the investor under consideration has 80 call options, he would be entitled to buy 80(100) = 8,000 shares. or 0.80 percent of all shares outstanding, at a total price of (8,000)($45) = $360,000, which represents 0.72 percent of the current company value of (1,000,000)($50) = $50,000,000. If a 2-for-1 stock split would occur, the share price would drop to $25. Without any adjustment, the 80 options would become worthless as the investor would better buy 8,000 shares at $25 a share, than exercise his 80 options at a price of $45 a share. However, if the occurrence of the stock split would allow him to buy double the original number of shares at half the original exercise price, he would buy (2)(80)(100) = 16,000 shares at a price of 45/2 = $22.50 per share. As before, he pays only (16,000)($22.50) = $360,000 or 0.72 percent of the current company value of (2,000,000)($25) = $50,000,000, to acquire 16,000/2,000,000 = 0.80 percent of all shares outstanding. Under these circumstances, there is no reason why, other things being equal, the value of his options would have changed in reaction to the stock split. But since the number of shares that option holder is entitled to buy has doubled (16,000 instead of 8,000 originally), the option value actually is cut in half.

Summing up, this example clearly shows that, other things being equal, the option value per share is halved whenever both the exercise price and the share price are halved. This property of the value of regular call options can be expressed in term of the value function defined in Equation 2.1: $f(S,K,T)/2 = f(S/2,K/2,T)$. The left side of the equation corresponds to half the original value (per share) of an option to buy shares at an exercise price K when the current share price is S while the right side equals to the value (per share) of an option to buy the same stock at half the original exercise price when the share price is halved as well.

By extension, it is easily seen that the following more general property applies:

$$\frac{f(S,K,T)}{\lambda} = f(S/\lambda,K/\lambda,T) \tag{2.2}$$

where λ represents any positive real number. Substituting K as a value for λ in Equation 2.2 to obtain: $f(S,K,T)/K = f(S/K,K/K,T)$. In view of Equation 2.1, this formula can be reduced to: $V_o/K = f(S/K,1,T)$. Finally, the right-hand side of this equation can be expressed as a value function f' of only two variables S/K and T as the following:

$$V_o/K = f'(S/K,T) \tag{2.3}$$

From this equation, it is clear that it is possible to omit the exercise price of a call option as a separate value per share, as long as both the option value and the stock price are expressed as a fraction of the exercise price indicated on the option. Let V_o' and S' denote these fractions, respectively, that is, if by definition:

$$V_o' = V_o/K$$
$$S' = S/K$$

Equation 2.3 can also be written as:

$$V_o' = f'(S',T)$$

2.2.1 Value of a Call Option at Expiration

In order to determine the per share value of a regular call option at expiration, one must find the specification of $f(S,K,0)$ for any given values of S and K. At the time of expiration, the option holder can do one of the following: either exercise the option or he can let the option expire, in which case it becomes worthless. By exercising the option, he would buy shares at a discount or at a premium relative to the current share price, if the current share price is greater or smaller than the exercise price. Obviously, the option holder would only choose to exercise the option if he can buy the shares at a discount (i.e., if the option is in-the-money). In this case, its value per share is equal to the difference between the current share price S and the exercise K. However, if the option is out of the money at expiration, this procedure would lead to a loss and it would therefore be better to let the option expire without exercise.

The above conclusion can be summarized as follows:

$$f(S,K,0) = \max\{0,(S-K)\} \tag{2.4}$$

Indeed, according to this formula, the option value per share at expiration is equal to (S - K) if $S > K$ (in the money), and to 0 if $S < K$ (out of the money).

The value of function $f(S,K,0)$, as determined by Equation 2.4, not only represents the value per share of an expiring option, but also the *intrinsic value* or *parity value* per share of a nonexpiring option. In addition, $f(S,K,T) - f(S,K,0)$, the difference between the full option value per share and the intrinsic value per share, is usually referred to as the option's *time value* per share.

To illustrate the above formula, consider an expiring option with an exercise price of \$30. If the current share price of the underlying stock is \$39, this option would be in the money, and therefore, its value per share would be equal to \$39 - \$30 = \$9. However, if the current share price is \$27, the option would be out of the money, and therefore, it would have a zero value. If the option were currently not expiring, these respective values of \$9 and \$0 could still be interpreted as the option's intrinsic value per share.

Equation 2.4 is graphically illustrated in Figure 2.1. The straight line with a slope of 45° that cuts the horizontal axis at K represents the difference between the share price S and the exercise price K, which corresponds to the profit per share earned when the option is exercised. As a negative profit or less can always be avoided by letting the option expire without exercising it, this difference only represents the option value per share if $S > K$, that is , only the part of the 45° line is relevant. If $S < K$ at expiration, the option's value per share is reduced to zero, and is therefore represented by a straight line that coincides with the horizontal axis. As a result. the option value per share at expiration for all possible stock price is represented in Figure 2.1 by the kinked straight line OKF.

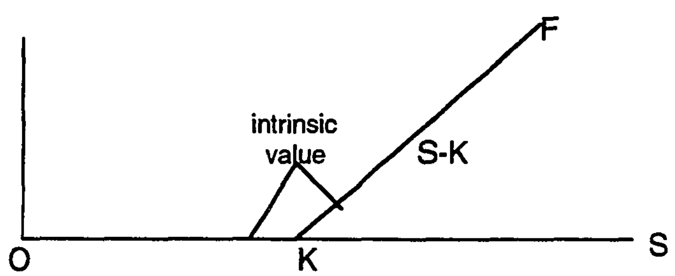

Figure 2.1: Intrinsic Value

Alternatively, by expressing the share price as a function of the exercise price, the option value per share can also be derived as a fraction of the exercise price from the following calculation formula: $f'(S/K,0) = \max\{0,(S/K)-1\}$, or, equivalently,

$$f'(S',0) \quad = \quad \max\{0,S'-1\} \qquad (2.5)$$

For example, reconsidering the preceding numerical example of an expiring option with an exercise price of \$30, if the current share price is \$39, a share of common stock would be worth 1.3 times the exercise price, and the option value per share would therefore be equal to 1.3-1=30 percent of the exercise price. At a current price of \$27, however, a share would be valued at only 90 percent of the exercise price, As this fraction in smaller than 1, the corresponding option value per share would be zero. If the option were not currently expiring, it would have an intrinsic value of 30 percent and 0 percent of the exercise price, respectively.

Equation 2.5 can also be represented by a kinked straight line, as shown in Figure 2.2, where the option value per share, as a fraction of the exercise price, corresponds to the broken line OK'F'.

In this subsection, only the regular call options at their maturity were considered. At maturity, the option holder can only choose between exercising the option and letting it expire. This choice was completely determined by the current share price relative to the exercise price.

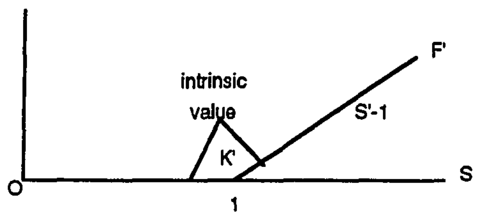

Figure 2.2: Intrinsic Value as a Fraction of Exercise Price

To consider options before their expiation date, however, additional factors must be taken into account, like the time value of money as measured by the risk-free rate of interest, and the investor's expectations about the unknown future share prices up to the time of expiration. The role of these other factors will be discussed in the next subsection.

2.2.2 Lower Limits on the Value of a Call Option Before Expiration

When considering a call option some time before expiration, the value of all the relevant parameters at the same point at the same tine must be carefully considered. Indeed, by the assumption that money has a positive time value, one cannot simply add or subtract monetary values of cash flows at different points in time. Only the equivalents of these cash flows at one and the same point in time can be compared.

In order to determine the current option value per share, therefore, the current share price should be compared to the present value of the exercise price, rather than to the exercise price itself. Indeed, if an option is exercised at future time, the exercise price also refers to a future cash flow resulting form a future stock purchase transaction. For example, by assuming that the option is exercised at expiration, the present value of the corresponding cash flow would be equal to $K(1+r)^{-t}$, where t represent the remaining time to expiration and r the applicable risk-free rate of interest. At least this would be its present value if the interest were compounded one per unit of time. In the case of continuos compounding, the present value is equal to Ke^{-rt}. In general, let this present value of the exercise price at expiration be denoted by PV(K,r,t). As this present value is proportional to K, it is also equal to K times PV(1,r,t), where PV(1,r,t) represents the present value of a cash flow of 1 dollar at expiration for a rate of interest r. PV(1,r,t) is often referred to as the present value interest factor at a specified rate for t time units. For example, the present value interest factor at 1 percent per month for 5 months would be equal to $(1+r)^{-t} = (1.01)^{-5} = 95.15$ percent if interest is compounded on a monthly basis, and it turns out to be equal to $e^{-rt} = e^{-.05} = 95.12$ if interest is compounded continuously.

If S > PV(K,r,t), the option holder could always realize a risk-free profit with a present value equal to S-PV(K,r,t) per share by short selling the stock at the current price S and buying it back at a price K and by exercising the option at its expiration. If S < PV(K,r,t), such an option would lead to a loss, which could, however, always be avoided by simply letting the option expire instead. This means that the option holder can always (1) avoid losing money and (2) realize a guaranteed minimum profit per share equal to S-PV(K,r,t) whenever the current stock price in high enough to make this latter expression positive. The option value per share, therefore, can never be smaller than (1) zero, and (2) S - PV(K,r,t):

$$V_o \geq \max\{0, S - PV(K,r,t)\} \qquad (2.6)$$

For example, if interest is compounded once a month at a risk-free monthly rare of 1 percent, and if the current price of a share of common stock is \$39, the value per share of an option to buy shares at an exercise price of \$30 within the next 5 months is smaller than \$39 - (95.15 percent)(\$30) = 10.55. As it was determined before, that the given share price of \$39 implies an intrinsic value of \$9, this also means that at 5 months to expiration the option's time value is not less than \$10.55 - \$9 = \$1.55 per share.

The lower limit on the option value per share described by Inequality 2.6 is sometimes referred to as the option's floor value. It is represented in Figure 2.3 by the kinked line OGH, while the intrinsic value per share is represented by OKF. Point G on the horizontal axis corresponds to the present value of the exercise price. Therefore, the floor

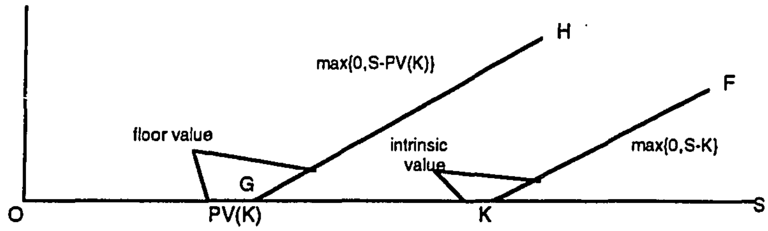

Figure 2.3: Option Floor Value

OGH is obtained by shifting the intrinsic value line OKF in a parallel way to the left over a distance equal to the difference between the exercise price and its present or discounted value.

2.2.3 Exercising an American Call Option Before its Expiration Date

From Figure 2.3 it is clear that, if at some time before maturity the stock price exceeds the discounted value of the exercise price, the option value is always strictly greater than its intrinsic vale. In particular, the value of any unexpired in-the-money call option always exceeds its intrinsic value. As the intrinsic value of such an option is the profit that can be realized by exercising it and selling the acquired stock immediately thereafter, it would be more profitable to sell that option than exercise it.

On the other hand, if an option is out of the money, its exercise price is by definition higher than the current stock price, and therefore, it would be cheaper to buy the stock directly than to buy it by exercising the option.

Summing up, an American call option, whether in the money or out of the money, should never be exercised before maturity[2]. This result may seem paradoxical, but can be explained by the fact that when exercising a call option, only the intrinsic value of that

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[2]There exists an exception to this rule, namely, it may sometimes pay to exercise a call option just prior to an ex-dividend date for the underlying stock. The reason is that when the option is exercised after that date, the option holder is no longer entitled to receive the dividend.

option is realized, but not its time value. This time value can only be realized by selling the option. Therefore the basic dilemma that any holder of an unexpired option is facing is not whether or not to exercise the option, but rather, whether or not to sell the option, where the alternative to selling is to let it mature. Furthermore, since American call options are never exercised before maturity, they give the option holder no real advantage over European call options. Consequently, the value of American call options must be identical to the value of corresponding European call options with the same characteristics.

2.2.4 Addition Properties of a Call Option Before Expiration

Whenever an option holder can realize a profit by selling stock at the present time at its current market price S and buying it back later at the exercise price K, the present value of that profit was shown previously to correspond to the option's floor value. When these transactions do not lead to a profit, that floor value is zero. The way to investigate how much the current market value of the option exceeds its floor value, is to examine at the same point in time for different stock price levels that cover a whole range, from very low to very high values of S. First, consider the case in which S is so small relative to the option's exercise price K, and that it is very unlikely that it will become greater than K before the option's expiration date, As it is virtually certain that such an option will remain out of the money until that date, it is said to be it far out of the money, In view of its very small profit potential, its market value will not significantly differ from zero; that is, it will not significantly differ from its floor value.

If, starting from such a low stock price level, S is gradually moved higher, the option value would leave its floor and gradually increase, even though, the floor value itself would remain zero as long as $S < PV(K,r,t)$. The reason is that even though the option cannot be profitably exercised under those circumstances, there is always a probability that the stock price may climb higher later on, and exceed PV(K,r,t) sometime before the option's expiration date. If and when this happens, a profit can be realized by selling the stock and exercising the option at maturity. This prospect of future gain opportunities, in combination with the fact that one can always avoid losses by letting the option expire, causes the option to have a positive value. The better the prospect of future gains is considered to be by investors, the greater this option value. In particular, as this prospect is better the smaller the difference between the stock price and the discounted exercise price PV(K,r,t), the option value will increase with increasing stock price levels. And, since the floor value is constantly equal to zero in the range of stock price levels below PV(K,r,t), the option's excess value over its floor value will increase in that range as well.

At this point, it is the time to determine what happens to that excess value when considering the further increased in the stock price level beyond the discounted value (PV(K,r,t)) of the exercise price. Beyond that level, risk-free profits could be realized by immediately selling stock at its current market price and repurchasing the stock by exercising the option at expiration. These profits are greater the higher the stock price, which causes the option's floor value to rise when higher stock prices are considered. At the same time, however, the prospect of making even bigger profits by delaying the stock sale starts deteriorating. On fact, in case of decline in the future stick price, such a delaying would lead to smaller rather than bigger profits. Even worse, if that decline causes the stock price to become smaller than the discounted value of the exercise price, selling the stock would no longer be profitable. Obviously, the greater the current price, the greater the potential decrease in profits, and therefore the less attractive the prospect resulting from a delay in the stock sale.

This phenomenon forces the option's excess value over its floor value to gradually diminish when increasingly higher current stock prices are considered beyond the discounted exercise price. For current stock prices that are very high relative to its exercise price, that excess value will virtually have vanished and the option is said to be *deep in the money.* Its value can, for all practical purposes, be considered to coincide with its floor value. In fact, at this point the option starts behaving in the same way as the underlying stock. Thus, the change in the stock price is reflected in a matching change in the per share value of the option.

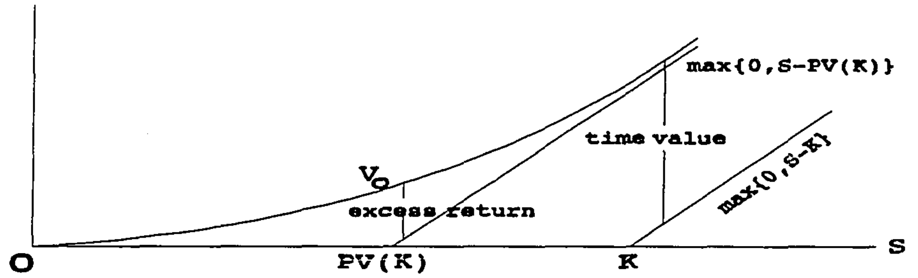

Figure 2.4: Option value

These results are graphically illustrated in Figure 2.4, where the curvilinear line V_o represents the option's market value per a share. As in the preceding Figures, the option's floor value and the intrinsic value per share are again represented by broken lines. As was shown by Merton (1973), the curve representing the option's market value per share will increase with rising stock prices below the discounted exercise price PV(K,r,t), but decrease with rising stock prices above that level. The maximum excess value therefore is reached when S = PV(K,r,t).

Figure 2.4 also shows that the intrinsic and floor values of an option coincide at the zero level for stock prices below the discounted exercise price PV(K,r,t). As a consequence, the total value of the option, the excess value over its floor value, and its time value all coincide for these stock price levels. However, if the stock price is greater than the discounted exercise price, the floor value is strictly greater than the intrinsic value, due to the time value of money by which the exercise price is discounted.

Furthermore, this difference between the floor value and intrinsic value must be added to the option's excess value to obtain its time value. The time value, therefore, has two distinct components. Gepts (1987) has pointed out that the option time value consists of: (1) the time value of money by which the exercise price is discounted, that is, the interest that can be earned by delaying the exercise of the option until its expiration date, and (2) The value derived form the prospect of possible sizable increases in realizable profits between now and the expiration date, in combination with only limited risks of profit decreases. Realizable profits go up or down together with the stock price. However, while they can increase over time in an unrestricted way, they can never turn into losses in view of the built-in zero-level floor, no matter how far the stock price would fall. Still, for deep-in-the-money options, the potential profit decreases and related risks are so great that they totally offset the chances for further gains. At this point, the second component of the option's time value vanishes, and that the total option value is reduced to its floor value.

2.3 The Black-Scholes Model

To this point, only the general shape of the course that describes the relationship between the option value and the stock prices has been determined. The specific numerical value of a call option can determined according to the Black-Shoes option valuation formula.

Black and Scholes (1973) demonstrated how this can be done under certain simplifying assumptions. The main assumptions are:

• The stock price follows a random walk and the continuously compounded return on the stock is normally distributed.

• The rate of interest is constant until the option's maturity date.

• There are no transaction costs, no taxes, and no penalties for short selling.

• The stock do not pay a dividend during the life of the option.

Their formula has gained wide recognition, and it is generally used by professional investors to estimate the value of options. Although the formula itself is not simple in its appearance, it can easily be applied, even by less experienced investors. Nowadays it is a common feature of investment packages for personal computers and it is also available to users of some of the more advanced programmable hand calculators. It specifies the option value per share V_o as estimated by the Black-Scholes formula is as follows:

$$V_o = SN(d_1) - PV(K,r,t)N(d_2), \tag{2.7}$$

where

$$d_1 = \frac{\ln(S/K) + (r+.5\sigma^2)t}{\sigma\sqrt{t}} \tag{2.8}$$

$$d_2 = \frac{\ln(S/K) + (r-.5\sigma^2)t}{\sigma\sqrt{t}} \tag{2.9}$$

The symbol N in Equation 2.7 represents the cumulative standardized normal probability distribution. That is $N(d_1)$ and $N(d_2)$ are the probabilities that a normally distributed random variable, with a mean of 0 and a standard d deviation of 1, does not take values greater than d_1 and d_2, respectively. The parameter r in the above formulas again represents the continuously compounded risk-free rate of interest, which implies that $PV(K,r,t) = Ke^{-rt}$ in Equation 2.7. Finally, the symbol σ^2, which appears in Equation 2.8 and 2.9, stands for the volatility of the stock price as measured by the variance per unit of time of the continuously compounded rate of return on the stock. It should be noted that the variance, and not its square root (the standard deviation) is proportional to the length of the time period over which it is defined. Therefore, when redefining the time unit, one should first adjust the variance in the same proportion, then take the square root to determine the adjusted value of the standard deviation. First, adjusting the standard deviation, and then raising the adjusted standard deviation to the power 2 to determine the variance, would definitely lead to incorrect volatility figures.

In total, the Black-Scholes model has five parameters, namely S, K, r, t, and σ^2. Cash dividend payments do not show up as a parameter because, strictly speaking, the formula only applies if no cash dividends are paid during the remaining life of the option. The volatility cannot be observed, as it reflects investors' expectations about the future behavior of the rate of return on the stock. It can, however, be estimated form historical stock price fluctuations, since the Black-Scholes model is derived under the assumption of an identically distributed rate of return over time. If this assumption in fact were strictly true over all periods, then estimates of the variance from historical data would be very good. As an example of this procedure, assume that an investor wishes to estimate the appropriate variance for some stock using one year of historical weekly data. The price relative for each stock is simply the price at the end of the week plus any dividends divided by the price at the beginning of the week. The natural logarithm of the price relative is the continuously compounded rate of return, and can easily be computed by applying the standard formula to the sequence of continuous compounded rates of return. For example, standard deviation is $\sqrt{\sum_{i=1}^{N}\left(\frac{(x_i-\overline{x})^2}{N}\right)}$, where x_i is the logarithm of the price relative. To convert the continuously compounded weekly standard deviation to a yearly standard deviation, simply multiply the square root of 52.

To illustrate the formula, the example from the previous section is used by assuming a risk-free rate of 1 percent per month, a current stock price of \$39, a remaining life of 5 months, and an exercise price of \$30. Using the symbols of this chapter, this means that r = 1 percent per month, that t = 5 months, that S = \$39, and K = \$30. In addition, let the stock's volatility σ^2 equal (28 percent $)^2$ = .0784 percent, or 0.0784 / 12 = 0.0065 per month. The values of parameters d_1 and d_2 in the Black-Scholes formula can be determined as follows:

$$d_1 = \frac{\ln(39/30) + 5[.01 + (.5)(.0065)]}{\sqrt{(.0065)(5)}} = 1.8228$$

$$d_2 = \frac{\ln(39/30) + 5[.01 - (.5)(.0065)]}{\sqrt{(.0065)(5)}} = 1.6425$$

One may wish to use the following polynomial approximation which may have been applied to create the standard normal tables. The cumulative normal probability can be computed by

$$N(x) \equiv 1 - \frac{1}{2\pi} e^{-0.5x^2} \left(b_1k + b_2k^2 + b_3k^3 + b_4k^4 + b_5k^5\right)$$

where $k = 1/(1+\rho x)$ and,

$$\begin{array}{ll} b_1 \equiv .31938153, & b_4 \equiv -1.821255978, \\ b_2 \equiv -.356563782, & b_5 \equiv 1.330274429\ , \\ b_3 \equiv 1.781477937, \text{ and} & \rho \equiv .2316419. \end{array}$$

This approximation produces values on $N(d_1)$ and $N(d_2)$ can be found from the area under the standard normal distribution from $-\infty$to d_1 , respectively:

$$N(d_1) \equiv N(1.82) = .9656$$

$$N(d_2) \equiv N(1.64) = .9495$$

which lead to the following option value per share:

$$V_o = SN(d_1) - Ke^{-rt}N(d_2)$$
$$= (\$39)(.9656) - (\$30)(e^{-.05})(.9495) = \$10.56$$

Apparently, the value of this option is only slightly higher than its previously determined floor value of $10.46. This option has an intrinsic value of $9 which can be concluded that its time value per share is $10.56 - $9 = $1.56. This time value consists of $30 (1 -$e^{-.05}$) = $1.46 of the time value of money by which the exercise price is discounted, and for the $0.10 excess value that reflects the degree by which the chances of making even higher profits from a further increase in the stock price outweigh the risk of a limited decline in profits that would result from a decrease in the stock price.

Some important conclusions can be drawn about the direction in which changes in the parameter values affect the option value. It can be shown that a call option is more valuable:

- the higher the current stock price relative to the exercise price,
- the higher the risk-free rate of interest,
- the longer the remaining time to expiration, and
- the greater the volatility of the underlying stock.

CHAPTER 3. THE BLACK-SCHOLES OPTION PRICING MODEL

In this chapter the famous Black-Scholes option pricing model will be derived. Although, the call option price is the solution to the general equilibrium pricing frame work, the Black-Scholes option pricing model can also be treated as the risk-neutral or preference-free pricing of options. However, the Black-Scholes formula can only apply to European call options. Therefore. it can only be used for valuing American call options if early exercise is most unlikely. Fortunately, the Black-Scholes formula can be generalized to include the case of American call options on dividend-paying stocks. This extension of the model will be discussed at the end of this chapter

3.1 Derivation

The general equilibrium pricing solution to the call option pricing problem incorporates the following assumptions:

- All individuals can borrow or lend without restriction at the instantaneous riskless rate of interest, r, and that rate is constant through the life of option T.
- The capital market is free from transaction cost (e.g., brokerage fees, transfer taxes, short selling and indivisibility constraints and tax differentials between dividend and capital gain income).
- The stock pays no dividends during the option 's time to expiration.
- Stock price dynamics have the return dynamics as

$$\frac{dS}{S} = \alpha\, dt + \sigma\, dz \qquad (3.1)$$

where α is the instantaneous expected rate of return, σ^2 is the variance rate which is assumed to be a constant, dz is the standard diffusion process (a Wiener process), having mean, and variance equal to zero , and one, respectively. In addition, it is assumed that the return dynamics on the option are descirbed by

$$\frac{dC}{C} = \alpha_c\, dt + \sigma_c\, dz \qquad (3.2)$$

where α_c and σ_c are the instantaneous expected rate of return and standard deviation of return on option, and C (S, t) is the option price.

Taking the Brownian motion, Equation 3.1, as given, the change in option value can be found by applying Ito's lemma as follows:

$$\begin{aligned} dC &= C_t\, dt + C_s\, dS + \frac{1}{2} C_{ss}\, dS^2 \\ &= \left(C_t + SC_s \alpha + \frac{1}{2} S^2 C_{ss} \sigma^2\right) dt + SC_s \sigma\, dz \end{aligned}$$

where $C_x = \dfrac{\partial C}{\partial x}$, and $C_{xx} = \dfrac{\partial^2 C}{\partial x^2}$. Comparing the above result to the assumed rate of return on option Equation 3.2, yields

$$\alpha_c C = C_t + \alpha S C_s + \frac{1}{2} S^2 C_{ss} \sigma^2 \qquad (3.3)$$

$$\sigma_c C = \sigma S C_s \qquad (3.4)$$

Consider forming a portfolio by investing a fraction γ in the option and 1 - γ in the stock. The return on this portfolio is

$$\begin{aligned} \frac{dP}{P} &= \gamma \frac{dC}{C} + (1-\gamma)\frac{dS}{S} \\ &= [\gamma\alpha_c + (1-\gamma)\alpha]dt + [\gamma\sigma_c + (1-\gamma)\sigma]dz \qquad (3.5) \end{aligned}$$

In order to have a riskless position on the portfolio, it is required that

$$\mathrm{var}_t\left(\frac{dP}{P}\right) \;=\; \mathrm{var}_t\left\{[\gamma\sigma_c + (1-\gamma)\sigma]dz\right\} \;=\; 0 \qquad (3.6)$$

where var_t denotes variance conditioned on (S(t), C(t)). To avoid the possibility of arbitrage, the expected rate of return on the portfolio must be equal to the risk-free rate of interest, that is

$$E_t\left(\frac{dP}{P}\right) \;=\; E_t\left\{[\gamma\alpha_c + (1-\gamma)\alpha]dt\right\} \;=\; r\,dt \qquad (3.7)$$

From Equations 3.6 and 3.7, substituting for γ and rearranging terms gives

$$\frac{\alpha - r}{\sigma} = \frac{\alpha_c - r}{\sigma_c} \quad . \qquad (3.8)$$

That is, the net rate of return per unit of risk must be the same for the two assets. Substituting in Equation 3.8 for α_c and σ_c from Equations 3.3 and 3.4, and collecting terms gives the Black-Scholes partial differential equation for option pricing:

$$\frac{1}{2}\sigma^2 S^2 C_{ss} + rSC_s - rC + C_t \;=\; 0 \quad . \qquad (3.9)$$

Its boundary condition is determined by the specification of the assets. For the case of an option which can be exercise only at the expiration date T with exercise price K, the boundary condition is

$$\begin{aligned} C(0,T) &= 0\,, \\ C(S,T) &= \max[0, S_T - K]\,. \end{aligned}$$

Subject to the boundary condition, the solution is given by Black-Scholes (1973) and Merton (1973) as the discounted value of the expected payoff

$$\begin{aligned} C(S,T) &= e^{-rT} E[\max(S_T - K, 0)\,|S] \\ &= e^{-rT}\int_K^{\infty}(S_T - K)\Pi(S_T\,|S)\,dS_T \quad , \qquad (3.10) \end{aligned}$$

where $\Pi(S_T|S)$ is the probability density function of the terminal stock price conditional on its current value S. For constant r and σ the stock price at T will be lognormally distributed with density function

$$\Pi(S_T|S) = \frac{1}{S_T\sigma\sqrt{T}} Z\left(\frac{\ln(S_T/S)-(r-0.5\sigma^2)T}{\sigma\sqrt{T}}\right)$$

where Z(.) is the unit normal density function. Substituting the density function into the option pricing function and evaluating the integral by using the fact that the truncated mean of a lognormally distributed random variable z with density function

$$\Pi(z) = \frac{1}{\sqrt{2\pi}\sigma z}\exp\left[\frac{(\ln z-\mu)^2}{2\sigma^2}\right]$$

is

$$E(z|z>a) = \int_a^\infty z\Pi(z)dz = \exp\left(\mu+\frac{\sigma^2}{2}\right)N\left[\frac{\mu-\ln a}{\sigma}+\sigma\right],$$

where N(.) is the cumulative distribution function of a standard normal random variable (.). Applying these two properties to Equation 3.10 yields

$$C(S,T) = e^{-rT}\left[e^{rT}SN(d_1)-KN(d_2)\right]$$

or

$$C(S,T;K,r,\sigma^2) = SN(d_1)-Ke^{-rT}N(d_2) \qquad (3.11)$$

where d_1 and d_2 are defined as,

$$d_1 = \left[\ln(S/K)+(r+0.5\sigma^2)T\right].\frac{1}{\sigma\sqrt{T}}$$

$$d_2 = d_1-\sigma\sqrt{T}$$

The numerical values of call prices at different exercise prices, K, time to expiration, T, and variance rates σ^2, for \$100 stock prices and 10 percent risk free rate of interest are shown in the Tables 3.1-3.6

Table 3.1: The Black-Scholes Option Values for 0.1 Standard Deviations at Various Maturities, Measuring Unit in One Year.

K	$\sigma = 0.1$						
	MATURITY (T)						
	.125	.25	.375	.50	.625	.75	.875
50	50.62	51.24	51.84	52.44	53.03	53.61	54.19
60	40.75	41.48	42.21	42.93	43.64	44.34	45.03
70	30.87	31.73	32.58	33.41	34.24	35.06	35.87
80	20.99	21.98	22.95	23.90	24.85	25.78	26.70
90	11.12	12.23	13.33	14.42	15.50	16.56	17.60
100	2.11	3.45	4.67	5.85	6.99	8.12	9.22
110	0.01	0.19	0.58	1.14	1.81	2.56	3.37
120	0	0	0.02	0.09	0.24	0.47	0.80
130	0	0	0	0	0.02	0.05	0.12
140	0	0	0	0	0	0	0.01
150	0	0	0	0	0	0	0

Table 3.2: The Black-Scholes Option Values for 0.2 Standard Deviations at Various Maturities, Measuring Unit in One Year.

K	$\sigma = 0.2$						
	MATURITY (T)						
	.125	.25	.375	.50	.625	.75	.875
50	50.62	51.24	51.84	52.44	53.03	53.61	54.19
60	40.75	41.48	42.21	42.93	43.64	44.34	45.03
70	30.87	31.73	32.58	33.42	34.26	35.09	35.91
80	21.00	21.99	23.01	24.03	25.03	26.04	27.02
90	11.25	12.65	14.00	15.29	16.53	17.72	18.87
100	3.47	5.30	6.86	8.28	9.61	10.88	12.09
110	0.44	1.17	2.60	3.74	4.88	5.99	7.10
120	0.02	0.27	0.77	1.42	2.17	2.98	3.83
130	0	0.03	0.18	0.46	0.86	1.35	1.92
140	0	0	0.03	0.13	0.31	0.56	0.90
150	0	0	0.01	0.03	0.10	0.22	0.40

Table 3.3: The Black-Scholes Option Values for 0.3 Standard Deviations at Various Maturities, Measuring Unit in One Year.

K	σ = 0.3						
	MATURITY (T)						
	.125	.25	.375	.50	.625	.75	.875
50	50.62	51.24	51.84	52.44	53.03	53.62	54.20
60	40.75	41.48	42.21	42.95	43.68	44.42	45.15
70	30.87	31.75	32.66	33.60	34.54	35.48	36.41
80	21.04	22.25	23.51	24.76	25.98	27.17	28.32
90	11.79	13.71	15.45	17.04	18.52	19.91	21.24
100	4.85	7.22	9.17	10.91	12.50	13.98	15.39
110	1.37	3.22	4.93	6.52	8.02	9.45	10.82
120	0.27	1.23	2.41	3.67	4.92	6.17	7.40
130	0.04	0.41	1.10	1.95	2.91	3.91	4.95
140	0	0.12	0.46	1.00	1.66	2.42	3.25
150	0	0.03	0.19	0.49	0.93	1.47	2.10

Table 3.4: The Black-Scholes Option Values for 0.4 Standard Deviations at Various Maturities, Measuring Unit in One Year.

K	σ = 0.4						
	MATURITY (T)						
	.125	.25	.375	.50	.625	.75	.875
50	50.62	51.24	51.85	52.47	53.09	53.72	54.56
60	40.75	41.50	42.29	43.12	43.96	44.81	45.67
70	30.89	31.91	33.03	34.19	35.34	36.47	37.57
80	21.24	22.88	24.52	26.08	27.56	28.97	30.31
90	11.25	12.65	14.00	15.29	16.53	17.72	18.87
100	6.25	9.16	11.52	13.58	15.45	17.17	18.79
110	2.54	5.12	7.34	9.34	11.17	12.89	14.51
120	0.86	2.66	4.49	6.25	7.94	9.55	11.10
130	0.25	1.29	2.65	4.10	5.56	7.00	8.42
140	0.06	0.60	1.52	2.64	3.84	5.09	6.36
150	0.01	0.27	0.85	1.67	2.63	3.68	4.78

Table 3.5: The Black-Scholes Option Values for 0.5 Standard Deviations at Various Maturities, Measuring Unit in One Year.

K	σ = 0.5						
	MATURITY (T)						
	.125	.25	.375	.50	.625	.75	.875
50	50.62	51.25	51.90	52.59	53.32	54.05	54.80
60	40.75	41.59	42.55	43.56	44.60	45.63	46.65
70	30.97	32.30	33.74	35.19	36.59	37.94	39.24
80	21.66	23.83	25.87	27.76	29.51	31.14	32.69
90	13.63	16.68	19.22	21.44	23.45	25.30	27.02
100	7.64	11.11	13.87	16.26	18.41	20.38	22.21
110	3.82	7.07	9.78	12.16	14.32	16.31	18.17
120	1.70	4.33	6.75	8.48	11.05	13.00	14.83
130	0.71	2.57	4.59	6.57	8.48	10.31	12.07
140	0.27	1.48	3.08	4.77	6.48	8.17	9.82
150	0.10	0.84	2.04	3.45	4.94	6.46	7.98

Table 3.6: The Black-Scholes Option Values for 0.6 Standard Deviations at Various Maturities, Measuring Unit in One Year.

K	σ = 0.6						
	MATURITY (T)						
	.125	.25	.375	.50	.625	.75	.875
50	50.62	51.29	52.06	52.90	53.78	54.68	55.57
60	40.78	41.82	43.03	44.30	45.56	46.80	48.00
70	31.16	32.92	34.74	36.49	38.15	39.73	41.23
80	22.26	24.99	27.44	29.64	31.66	33.52	35.26
90	14.73	18.34	21.26	23.80	26.08	28.15	30.08
100	9.03	13.05	16.22	18.94	21.36	23.57	25.61
110	5.16	9.05	12.21	14.97	17.44	19.70	21.80
120	2.76	6.14	9.11	11.77	14.20	16.44	18.55
130	1.41	4.10	6.74	9.22	11.54	13.72	15.79
140	0.68	2.70	4.96	7.20	9.37	11.46	13.45
150	0.32	1.75	3.63	5.62	7.61	9.57	11.47

From Tables 3.1 through 3.6, all else equal, the higher the exercise price the lower the call price. This relationship can be investigated by considering calls with the same expiration date written on the same stock. The one with the higher exercise price cannot be more valuable than the one with lower exercise price, since the holder of the former, upon exercise, will have more cash left over. An increase in the interest rate or the expiration date will have the same effect to the option price. They will decrease the present value of the exercise price and thus, increase the call value. An increase in the variance rate of stock return will correspond to the extreme stock price movements. The chance that stock price levels will be greater than the exercise price will occur more often. And, since the value of option only counts for the positive value of the stock price less exercise price. The option value will be higher for the higher variance rate of return. Equation 3.9 can be written as

$$\frac{\alpha S C_s + C_t + \frac{1}{2}\sigma^2 S^2 C_{ss}}{C} - r \;=\; \frac{S C_s(\alpha - r)}{C}$$

Substituting from Equation 3.3 gives the relationship between the option's and the stock's expected rate of return

$$\alpha_c - r \;=\; \frac{S C_s(\alpha - r)}{C}$$

To get the option's standard deviation, substitute from Equation 3.8

$$\sigma_c \;=\; \frac{S C_s}{C}\sigma \quad . \tag{3.12}$$

The term $\frac{S C_s}{C}$ is the call option's price elasticity

$$\eta_c \;=\; \frac{S C_s}{C} \;=\; \frac{S N(d_1)}{S N(d_1) - K e^{-rT} N(d_2)}$$

The call option's price elasticity is always greater than one, in other words, from the relationship given in Equation 3.11 the option is riskier in percentage terms than the stock.

The construction of the Black-Scholes option formula is based on the hedge portfolio was discussed earlier. The portfolio consists of investing a fraction γ in the option and $1 - \gamma$ in the stock. This fraction, γ which is the fractional investment in option, is the solution to equation 3.6. That is,

$$\gamma = \frac{\sigma}{\sigma - \sigma_c} = \frac{C}{C - SC_s} \quad . \tag{3.13}$$

The portfolio constructed in Equation 3.5 consists of selling (buying) γ options and buying (selling) $1 - \gamma$ stocks such that the portfolio position is riskless with, r, rate of return. On the other hand, the portfolio that duplicates the option, replicating portfolio, can be constructed in the same manner. It is the combination of stock and the riskless asset that provides the same return as the option. Solving Equation 3.5 gives,

$$\frac{dC}{C} = \frac{\gamma - 1}{\gamma}\frac{dS}{S} + \frac{1}{\gamma}\frac{dP}{P} = \frac{\gamma - 1}{\gamma}\frac{dS}{S} + \frac{1}{\gamma} r\,dt \; .$$

Substituting the value of γ from Equation 3.13 into the above equation and rearranging terms,

$$dC = N(d_1)dS + \left(C - SN(d_1)\right) r\,dt$$

or

$$dC - N(d_1)dS = \left(C - N(d_1)S\right) r\,dt \tag{3.14}$$

This is the way offered by Black-Scholes (1972) to adjust for changing risk of a position in the option. Since the stochastic process of option prices is generated from, and thus dependent on, the stochastic process of the underlying stock prices, Black and Scholes suggest offsetting the former uncertainty with the latter. This can be done by creating a neutral hedge position composed of buying (selling) one option and, at the same time, selling (buying) a fraction of the underlying stock. This fraction, the hedge ratio, is determined in such a way as to cancel, over a very short time period, the risk of the option position with that of the equity position. The hedge ratio is, in fact, the partial derivative of the call option prices with respect to the stock price, $N(d_1)$. The left-hand side of the Equation 3.14 is the return on the portfolio, and the right-hand side is the riskless opportunity cost on the investment in the portfolio.

Since the value of d_1 is equal to

$$d_1 = \frac{\ln(S/K) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} ,$$

$N(d_1)$ can take on a value from zero to one, and the number of shares of stock per option will change as a function of changes in the stock price S and the duration of the contract, T. As T goes to zero, d_1 will approach minus or plus infinity depending on whether the stock price is below or above the striking price. As at expiration of the contract the number of shares of stock per option will be either zero or one. Also if the stock price increases, a larger fraction of a share will be needed to rebalance the option position, and if the stock price decreases a smaller fraction of a share will be needed.

3.2 Impact of a Dividend

Strictly speaking, the Black-Scholes formula discussed previously only applies to European call options. Therefore, it can only be used for valuing American call options if early exercise is most unlikely. Such is not the case, however, when the underlying stock is expected to pay dividends during the life of the American call option. The reason is that the option holder is not compensated for any dilution of the stock's value due to cash dividend payments. As a consequence, early exercise of the option may pay if a large enough dividend payment is expected soon enough. Fortunately, the Black-Scholes formula can be generalized to include the case of American call options on dividend-paying stocks. The extended model was originally developed by Richard Roll (1977), later simplified by Geske (1979), and finally perfected by Whaley (1981).

Let the stock's dividend history be described by the following additional assumptions:

- D a dividend of unknown size, will be paid to each shareholder with certainty.
- t is the known time until the ex-dividend instant ($t < T$). At t, the stock has just gone ex-dividend.
- α is the known decline in the stock price at the ex-dividend instant as a proportion of the dividend.
- No other dividend will be paid before T has elapsed

Roll (1977) has defined the stock price, S, as the total market price P, less the discounted escrowed dividend; i.e., for any $\tau < t$, $S_\tau = P_\tau - \alpha D e^{-r(t-\tau)}$ and for $\tau \geq t$. As a consequence, for the stock that pays a certain dividend, D, at the ex-dividend instant, t ($t < T$), the stock price simultaneously falls by a known amount, α D.

It is worthwhile to note that the stock price is assumed to drop by an amount α D rather than D at the ex-dividend instant. The coefficient, α, is the proportion of the dividend that the stock price may take by an amount other than the dividend amount. Occasionally, for example, it has been argued that the stock price is less than price appreciation.

At the instant before the stock goes ex-dividend, if the American option holder exercises his option, he will receive $S_t + \alpha D - K$, where S_t is the ex-dividend stock price. An instant later, if he allows it to remain unexercised, his option would be worth $C(S_t, T - t, K)$, where C (.) is the European call option formula. There exists some finite ex-dividend stock price, S^*, above which the American option will be exercised just before t if

$$S_t + \alpha D - K \quad > \quad S_t - Ke^{-r(T-t)}$$

$S_t - Ke^{-r(T-t)}$ is the lower bound for F $((S_t, T - t, K)$. The stock price S^* is the solution to

$$C(S^*, T-t, K) \quad = \quad S^* + \alpha D - K$$

Actually, the function $S_t + \alpha D - K$ and $C(S_t, T - t, K)$, do not need to have an intersection, if the income from early exercising the call is less than the lower boundary condition. In this situation, the value of the American call is $C(S', T, K)$, where $S' = S - \alpha De^{-rt}$, as pointed out by Whaley (1986).

If S^* exists, the value of an unprotected American call option is determined by the combination of

1) a long position of one European call option with exercise price K and maturity T;

2) a long position of one European call option with exercise price S^* and maturity $t - \varepsilon$; and

3) a short position in one Europe call on the option described by (1) with exercise price $S^* + \alpha D - K$ and maturity $t - \varepsilon$.

The above portfolio was constructed by Whaley (1981) using the duplication technique proposed by Roll (1977). If the ex-dividend stock price is above S^*, options (2), and (3) are exercised providing a net cash flow of $S_t + \alpha D - K$. Thus, the income contingencies of this portfolio at the instant after the ex-dividend date are equal to those posed by the American call option which can be summarized in Table 3.7.

Under the assumption that the stock price net of the escrowed dividend, S, is described by the stochastic differential equation

$$\frac{dS}{S} \quad = \quad \mu\, dt + \sigma\, dz$$

Table 3.7 Cash Flow of Compound Options

For $S_t > S_t{}^*$	For $S_t < S_t{}^*$
Cash receipts are	Portfolio positions are
From (1) 0	From (1) Open
(2) $S_t - S_t{}^*$	(2) Expired
(3) $S_t{}^* + \alpha D - K$	(3) Expired
Total $S_t + \alpha D - K$ in cash	Total Option on S until T with exercise price K

the Black-Scholes (1973) option pricing formula can be applied to options (1) and (2) of the portfolio. Let $N_1(A)$ be the univariate cumulative normal density function with upper integral limit A and $N_2(A,B;\rho)$ be the bivariate cumulative normal density function with upper integral limits A and B and correlation coefficient ρ. The Geske (1979)'s compound option pricing formula can be applied to option (3). The value of an American call option on a stock with a single dividend paid during the life of the option is

$$\begin{aligned} \Lambda(S,T,K) &= S\left[N_1(B_1) + N_2(A_1,-B_1;-\sqrt{t/T}\right] \\ &\quad - Ke^{-rT}\left[N_1(B_2)e^{r(T-t)} + N_2(A_2,-B_2;-\sqrt{t/T}\right] \\ &\quad + \alpha De^{-rT}N_1(B_2) \end{aligned}$$

where

$$\begin{aligned} A_1 &= \frac{\ln(S/K) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} \\ A_2 &= A_1 - \sigma\sqrt{T} \\ B_1 &= \frac{\ln(S/S^*) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} \\ B_2 &= B_1 - \sigma\sqrt{t} \end{aligned}$$

CHAPTER 4. A SURVEY OF EMPIRICAL TESTS

The amount of work devoted to empirical testing of call option pricing is rather impressive. While various approaches have been applied to different data sets and time periods, empirical research has met strong obstacles. The tests in the area of option pricing are not the most efficient, accurate, and conclusive. However, they have paved the way to better understanding not only of the behavior of option prices but also of stock prices.

The empirical research on options has revealed so many aspects. First of all, the studies helped to demonstrate the importance of the data accuracy, showing that even slightly inaccurate data can substantially affect results. Second, the empirical research on stock markets may have suffered from many problems that were ignored in the past. For example, the question of the stationary of the distribution of stock-price changes as well as the problems associated with the market microstructure have been reexamined in the option literature. Third, while the tests in almost any empirical study are joint, this issue has been stressed in the option pricing area more than in any other area in finance.

Various approaches for testing the validity of an option-pricing model will be discussed in the next section of this chapter. The main concern in testing option model based on the accuracy of the data collection which may reflect the efficiency of option market. The markets in which trading in related assets takes place simultaneously and quoted prices reflect this simultaneity are referred to as synchronous markets. Synchronization, therefore, has two facets: trading synchronization and data synchronization. The former stands for parallel trading in two related securities. Trading synchronization is not sufficient for proving market synchronization since data recording may be nonsynchronized. The technology for registration of trades must be such that data accurately present timing of the transaction and the time the information is made available to market participants. If data on a class of options and the underlying stocks are used and the price quotes are not taken at the same time, based on parallel trading, the markets will be nonsynchronous.

4.1 Testing Model Validity

Most of the discussion will concern testing the validity of the Black-Scholes model for pricing of call options. The following sections deal with different approaches to testing model validity. A survey of the literature reveals four major approaches for testing model's validity. These approaches will be outlined briefly in this section, while empirical results will be described later.

The first approach, initially suggested by Black-Scholes (1972), is based on creating neutral hedge positions, and testing the behavior of the returns from the investment. The idea is to create, with options and their underlying stocks, a position that, if the model is correct, should be riskless. By doing so, the problem of risk-adjustment for investment in option is eliminated.

The second approach is based on imputing the standard deviations from actual option prices by using a pricing model. It is assumed that all parameters except for the risk measure of the underlying stock, are known and that markets are efficient and synchronous. By equating the actual price to the model price, the standard deviation can be imputed as the only unknown in the equation. The behavior of the implied standard deviation is then investigated to determine the validity of the assumed model.

The third approach is based on direct comparison of the model prices to actual prices. According to this approach, the estimated parameters of the model and the actual observations of stock prices are placed in the pricing model to generate expected option prices. In the second stage the model prices are compared to the actual, realized option prices. The tests are intended to show whether model prices are unbiased estimators of actual prices or whether there are consistent deviations that can be exploited for better prediction or for making above-normal profits[1]. The major disadvantage of this approach is that the statistical characteristics of the time series of model prices or the deviations between them and actual prices are nonstationary. The risk characteristics change with time to maturity and with changes in stock prices.

The fourth approach for testing pricing models is by means of simulations of deviations from the basic assumptions of the models. According to this approach the sensitivity of the model prices to empirical deviations from the assumptions is tested. Actually, simulations can not be used for a direct test of model validity; they should be used for examining the robustness of the given model to varying conditions.

The majority of empirical papers use the first, second or third approaches, and most of the discussion that follows will be devoted to them.

4.2 Risk-Adjusted Returns

Most of the discussions that will be explored in the next section, ignore the risk associated with an option position. For instance, the stochastic nature of the deviation of the model prices from the actual prices, $C^a - C^m$, in MacBeth and Merville's study may change from day to day and across the underlying stocks. Moreover, the uncertainty is also a function of the extent to which the option is in-the-money. Hence the appropriateness of using a simple regression technique may be questionable.

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[1]For a complete riskless strategy, above normal profits mean profits in excess of the risk-free rate of interest.

It is one of the basic principles of modern finance theory that investment evaluation should include consideration of the risk associated with the return on the investment. To find the measures of risk is much more complicated, especially when contingent assets are considered. Since the distribution of the rates of return from an investment in an option is not expected to be symmetrical and stationary over time, simple measures of risk, such as standard deviation of the distribution may not be suitable.

In their empirical paper, Black-Scholes (1972) offer a way to adjust the changing risk of a position in an option. Since the stochastic process of option prices is generated from, and thus dependent on, the stochastic process of the underlying stock prices, Black-Scholes suggest offsetting the former uncertainty with the latter. This can be done by creating a neutral hedge position composed of buying (selling) one option and, at the same time, selling (buying) a fraction of the underlying stock. This fraction, the hedge ratio, is determined in such a way as to cancel, over a very short time period, the risk of the option position with that of the equity position. That fraction is, in fact, the partial derivative of the call option price with respect to the stock price, $\partial C / \partial S$, then for a very short time interval, Δt, the investment in an option (on one share), while short selling a fraction $\partial C / \partial S$ of the share, should be riskless. It is therefore expected that for small Δt,

$$\Delta C - \frac{\partial C}{\partial S}\Delta S = \left(C - \frac{\partial C}{\partial S}S\right) r \Delta t$$

where the Δ is the change operator. The left-hand side of the above expression is the return on the portfolio, and the right-hand side is riskless opportunity cost on the investment in the portfolio.

The procedure for creating the hedge position on day t is :

- compute the model price C_t^m and compare it with the actual price C_t^a.
- If $C_t^m < C_t^a$ the option is considered to be overvalued; it should be sold and

$\partial C / \partial S$ units of the underlying share should be bought.

- If $C_t^m > C_t^a$ the option is considered to be undervalued; the action that should be taken, is to buy the call and sell short $\partial C / \partial S$ units of share.
- The hedge position will be held over a time interval, and assumed to be liquidate on t+1.

The ex-post hedge return based on the Black-Scholes is thus,

$$R_{h,t+1} = (C_{t+1}^a - C_t^a) - N(d_{1t})(S_{t+1} - S_t)$$

for the undervalued call option, and

$$R_{h,t+1} = N(d_{1t})(S_{t+1} - S_t) - (C_{t+1}^a - C_t^a)$$

for overvalued call options. The hedge return minus the opportunity cost on the investment will give the excess-dollar return for the hedged position.

Black-Scholes (1972) calculated the excess-dollar return for options traded during the period May 1966 to July 1969 from the diaries of an option broker who had recorded all option contracts written for his customers. The diaries contained the striking price, the expiration date, the written date, the premiums received, the actual date of exercise, and the number of contracts written. By comparing their model prices with the actual ones at the issuing date of an option, they could classify options into two groups-those the market overvalued compared with the model, and those it undervalued. The initial position in the underlying stock was adjusted on a dairy basis to maintain the neutral hedge. On theoretical grounds, they expected the excess return to have zero systematic risk.

The major finding of the Black-Scholes' study was that by using past data to estimate the variance, the model overpriced options on high-variance securities and underpriced options on low-variance securities. This may indicate that traders underestimated variances for volatile stocks and overestimated them for stocks with low volatility. By also using ex-ante estimates of the variance based on data for life of the option they concluded that "if the model has an accurate estimate of the variance it works very well" (p. 416). In addition they observed nonstationary in the variance. the results may indicate market inefficiency. But, the profits from selling options on low-variance securities and buying options on high-variance securities completely disappeared when transaction costs were considered.

A secondary market in options was virtually nonexist in the over-the-counter market. Thus, while stock prices in Black-Scholes study were actual daily closing prices, the option prices they used in their tests were imputed for the model. As a result, their average excess-hedge return mainly reflected the difference between the option's actual price and the model price on the initial day. Because model prices instead of actual prices were used after the initial day, the relative changes in the underlying stock quite closely matched those imputed for the option during life of the option, and the excess-dollar return for these days was very close to zero.

In Galai (1977), the option positions are adjusted on a daily basis, and actual option prices are used. The data employed are the daily closing prices of all options traded during the first months of the CBOE, from 26 April 1973 to 30 November 1973. The total number of observations on option prices was 16,327. The ex-post test was performed under the assumption that trading at the closing price on day t, based on a trading rule that had been decided by using the same price, was possible.

The average hedge dollar return over 202 option classes, traded during the initial period of the CBOE, was approximately $9.80 per option day. For 71 options out of 202, the average of the time-series hedge return was significantly different from zero (at 5 percent). The results are thus consistent with the results shown for the boundary conditions in Galai (1978); the ex-post hedge strategy effectively located deviations from the model price, generating substantial 'book profit'. During the sample period the average hedge returns were stable across maturities. Regressing the hedge returns on the market index yield estimates of beta. There were in most cases, not significantly different from zero, while the estimates of the intercept (that is, the return adjusted for risk) were all significant (at 1 percent). The results were quite robust with respect to various estimates of the riskless interest rates and the variance of the rate of return. The latter was estimated in different ways from time series of past rates of return. By imposing a 1-percent transaction cost on changing the daily investment, almost all the hedge returns were eliminated.

The effect of dividend payment on the valuation of CBOE options was tested by classifying stocks in four groups according to their dividend yields and repeating the regression analysis for each group. The results clearly indicate that the ability of the unadjusted Black-Scholes model to locate over- or undervalued options declined as the dividend yield increased. The model performed better when applied to situations in which the basic assumptions of the model were better maintained, such as without-dividend payment. This conclusion was supported by the results obtained when the tests were limited to the trading days after the last expected ex-dividend day for any option. Finally, a simple adjustment for dividends was introduced to the Black-Scholes model by subtracting the present value of the expected dividends during the life of the option from the price of the underlying stock[2]-and using this adjusted price in the model. The hedge returns were substantially higher, by 50 percent on the average, after introducing the dividend adjustment.

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[2]Then the value of the option on a stock with current market price of S for which dividend D_{ti} is expected to be paid on day t_i will be approximately

$$C_t(S, D_{t1}, \ldots, D_{tn}) = \left(S - \sum_{i=1}^{n} e^{-rt_i} D_{ti}\right) N(d_1) - Ke^{-rt} N(d_2)$$

where $d_1 = \left\{\ln\left[\left(S - \sum_{i=1}^{n} e^{-rt_i} D_{ti}\right) / K\right] + (r + 0.5\sigma^2)t\right\} / \sigma\sqrt{t}$.

The dividend correction increased the ability of the hedging strategy to locate profitable opportunities.

Galai's study emphasized that to test market efficiency, an ex-ante test simulating the trading opportunities for the trader should be performed. The meaning of efficiency in finance literature is not in terms of Pareto optimality which, in the context of the option market, is the question of whether creating a new market place for trading options increases the welfare of society. In stead, a market will be termed efficient if no trader can consistently make above normal, risk-adjusted profits on an after-transaction cost and after-tax basis. The procedure used was similar to the one used in the ex-post tests, except that in the ex-ante test the execution of trading is delayed by one trading day. On day t it is decided whether the option is over or undervalued and the hedge ratio is calculated; the hedge is established on day t+1 and liquidated on t+2. The ex-ante hedge return is

$$R_{h,t+2} = \left(C^a_{t+2} - C^a_{t+1}\right) - N(d_{1t})\left(S_{t+2} - S_{t+1}\right)$$

if $C^a_t < C^m_t$; and

$$R_{h,t+2} = N(d_{1t})\left(S_{t+2} - S_{t+1}\right) - \left(C^a_{t+2} - C^a_{t+1}\right)$$

if $C^a_t \geq C^m_t$. Basing the analysis on the assumption that the trader has to wait (at least) 24 hours before he can execute his desired transactions, the average of the hedge returns fell from $9.80 to $5.00 per option per day for the ex-ante test. In addition, the dispersion of the returns increased substantially, and, hence, the number of averages that are significantly different from zero declined with the dividend yield from $7.30 per contract for the low-dividend-yield portfolio to $2.40 for high-dividend yield portfolio.

The evidence of the ex-ante test shows that profit opportunities might still have existed in the CBOE for the market to exploit. These profits were lower and perhaps riskier than those that seemed possible at first glance. Profits may be completely eliminated if a spreading technique is considered as suggested by Phillips and Smith (1980).

Market makers, especially, on the CBOE mainly use the spreading-option strategy which consists of a long position in one option and a short position in another, usually on the same underlying stock. Spreading can be performed on striking prices by buying and selling options that have the same expiration date but differ only in their striking prices. Also, spreading can be done on expiration date by buying and selling options that have the same striking price but differ in their maturities. Mixture of the two strategies can be constructed and are carried out on the exchange.

Let $N(d_{1j})$ be the hedge ratio for hedging option j with its underlying stock and $N(d_{1k})$ be the hedge ratio for option k written on the same underlying security. Another way to describe these relationships is that an expected riskless position can therefore be created by buying (selling) $1/N(d_{1j})$ units of j and selling (buying) $1/N(d_{1k})$ units of k. Equivalently, the riskless position can be achieved by buying (selling) one contract of option j and selling (buying) $N(d_{1j})/N(d_{1k})$ contracts of option k.

To determine whether j is over- or undervalued relative to k, Galai suggests comparing the ratio of the model prices C^m_{jt}/C^m_{kt} with the ratio of the actual prices C^a_{jt}/C^a_{kt}. If $C^m_{jt}/C^m_{kt} > C^a_{jt}/C^a_{kt}$ then the riskless position can be accomplished by buying option j and sell k because option j is underpriced relative to k, otherwise selling j and buying k.

If j is undervalued relative to k, the investment in day t will be

$$C_{jt} - \frac{N(d_{ijt})}{N(d_{1kt})} C_{kt}$$

and the return in t+1, R_{t+1} is,

$$R_{t+1} = \left[C_{jt+1} - \frac{N(d_{1jt})}{N(d_{1kt})} C_{kt+1}\right] - \left[C_{jt} - \frac{N(d_{1jt})}{N(d_{1kt})} C_{kt}\right]$$

$$= \left(C_{jt+1} - C_{jt}\right) - \frac{N(d_{1jt})}{N(d_{1kt})}\left(C_{kt+1} - C_{kt}\right)$$

where all the prices above are actual premiums. For the case in which j is overvalued relative to k, the return is

$$R_{t+1} = \frac{N(d_{1jt})}{N(d_{1kt})}\left(C_{kt+1} - C_{kt}\right) - \left(C_{jt+1} - C_{jt}\right)$$

Galai's study shows that the average spreading return for all options was $8.40 for ex-post spreading for July 1973 maturity against October 1973 maturity, and only $4.80 for the ex-ante[3] strategy.

While the Black-Scholes model assumes continuous trading, in practice trading is discrete, with the exchanges open for business for a limited time period each day. Galai (1983) studies the effect of this deviation from the basic assumption of the Black-Scholes model. In his article the profits from the hedging strategies with options are further analyzed. He shows that the results obtained for the ex-post and ex-ante hedge returns are not due to the discreteness of trading. The major component in the hedge return is the change in the deviation between the model and the market prices, while the opportunity cost and the discrete daily adjustment are marginal.

Chiras and Manaster (1978) calculate the spread returns for positions that indicate high profit potential. They use the Black-Scholes model with an adjustment for the expected dividends, and the weighted implied standard deviation[4] (WISD) as an estimator of the

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[3] if j was undervalued relative to k, the return on day t+2 when the position is liquidated is

$$R_{t+2} = \left(C_{jt+2} - C_{jt+1}\right) - \frac{N(d_{1jt})}{N(d_{1kt})}\left(C_{kt+2} - C_{kt+1}\right)$$

and if j was overvalued, it is

$$R_{t+2} = \frac{N(d_{1jt})}{N(d_{1kt})}\left(C_{kt+2} - C_{kt+1}\right) - \left(C_{jt+2} - C_{jt+1}\right)$$

[4] They use the price elasticity of an option with respect to its implied standard deviation as the weight.

standard deviation. A spread position is established for those options that deviate by 10 percent from the market price; that is,

$$\frac{C^m - C^a}{C^a} > 10 \text{ percent.}$$

The value of each option in the portfolio is determined by its hedge ratio. All option positions are maintained over one month. During twenty-two holding periods, from June 1973 to April 1975, 118 positions were formed and 93 of them were found to show paper profits. The average gain per position was $9.96 per month.

Chiras and Manaster claim that the results indicate market efficiency. Moreover, they are careful to note the ex-post nature of their tests and the potential problem of nonsimultaneity of option prices. Bookstaber (1981) used the data of Chiras and Manaster to check the effects of potential nonsimultaneity in their data. His conclusion is that there is strong support of the concern "that the observed profits were due to the noncontemporaneous data, and are not achievable in practice" (p. 155). In addition, Phillips and Smith (1980) show that by introducing transaction costs, especially those contained in the bid-ask spread, the profits of Chiras and Manaster are eliminated.

The bid-ask spread is the difference between the highest quote to buy and the lowest offer to sell the registered security in the market. These recorded offers come from two sources: (1) quotes from market makers/specialists, and (2) customer's limit orders recorded on the exchange's limit order book. The bid-ask spread contains implicit trading costs, especially, information costs in the option markets. Phillips and Smith observed that the average spread for stock is less than 1 percent, while the average spread for call options is 30 percent and for puts 15 percent. The average dollar spreads for stocks and options are closer, between $16 and $20.

In Galai's ex-ante hedging test, he establishes the hedge at closing prices from the next day's trading. By adjusting prices daily, Galai's ex-ante hedging test generates an abnormal return of $5 per hedge per day before transaction costs. However, as reported by Phillips and Smith, the average bid-ask spread for a call contract is $16 and for a round lot of stock is $20; thus the estimated bid-ask spread for the call alone more than eliminates Galai's $5 average profit. Moreover, Galai's spreading strategy, where call options which differ only in expiration dates are simultaneously bought and sold by buying undervalued and writing overvalued calls, produces ex-ante average of $4 which will be offset by estimated bid-ask spread of $16.

Blomeyer and Klemkosky (1982) test the validity of Roll's model for pricing the unprotected call option. They investigate to what extent the observed deviations of the Black-Scholes model prices from actual prices are due to the fact that the model ignores the early exercise opportunity of the unprotected American call. The data used in the study are transactions data for eighteen stocks and their CBOE options for one trading day per month during the period July 1977 to June 1978. Blomeyer and Klemkosky follow Galai's testing procedure, repeating the ex-post performance test for both the Black-Scholes and Roll models and comparing the average percentage-excess hedge returns generated by each model. The hedge positions are adjusted for each new option transaction price, and the returns are calculated over the intervals between successive transactions. The average reported for each underlying stock and for each trading day are also aggregated for three groups according to their dividend yields.

It is expected that since the Roll model allows for the early exercise of CBOE options, it should outperform the Black-Scholes model for options written on high-dividend-yield stocks. The results do not support the expectations, according to Blomeyer and

Klemkosky. They claim that the ex-post results suggest that the Roll model is not superior to the Black-Scholes model in its ability to identify overvalued and undervalued call options.

While no significant differences are found between the results based on the Roll model and those based on the Black-Scholes model, the ex-post excess-hedge returns are significantly different from zero
(at 5-percent significance level) for both models over most trading days and underlying securities. To test for market efficiency an ex-ante hedging strategy is constructed : a position is established based on the prices of the option and the stock approximately five to fifteen minutes after a substantial deviation between the model and the market prices is observed. The position is held over one month and then is assumed to be liquidated. The results are also adjusted for transaction costs[5] for relatively efficient traders as suggested by Phillips and Smith (1980). On a before-transaction-cost and opportunity-cost basis they find that the two models produce significant positive returns, but these average profits disappear after the adjustments are made. From average excess monthly returns over all days and securities of 1.2 to 1.6 percent, the excess return fell to -1.1 to -1.0 percent after adjusting for transaction costs. The results thus support market efficiency. Blomeyer and Klemkosky also conclude that the Black-Scholes model is an acceptable pricing alternative to the mathematically complex Roll model.

However, it should be noted that Blomeyer and Klemkosky use the average percentage-excess returns, which they get by dividing the excess-dollar return by the absolute investment, $C - \partial C / \partial S$. One problem may arise due to the size the investment can take; in some cases it may get very close to zero and the rate of return will be very high. One extreme value can significantly affect the average and its standard deviation. Another problem may result from using the weighted implied standard deviation (WISD) based on

[5] Blomeyer and Klemkosky use one-half of the mean bid-ask spread as the correct transactions cost since the trading rule use in their study does not dictate that transactions occur on the wrong side of the spread.

the Black-Scholes model. If the Roll model is correct , the Black-Scholes model prices will give an underestimation of the American call prices, and therefore, the Black-Scholes implied standard deviation will be overestimated. Using these WISD's in the Roll model will bias the prices upward. This problem will be especially serious for the high-dividend-yield stocks.

4.3 Implied Standard Deviation

One of the basic assumptions of the Black-Scholes model is that the standard deviation of the stock's rate of return is known and constant. If the model is correct and option, and stock markets are efficient and synchronous, by equating the model price to the actual market price the standard deviation implied by the option price can be imputed. It is expected, therefore, that the implied standard deviation (ISD) be stationary over time, across maturities and striking prices. The ISD is also expected, given the assumptions of the model, to be equal to the time-series standard deviation. Deviations from expectations may be due to markets' inefficiency or lack of synchronization among related markets, or to the model being misspecfied.

The estimation of the standard deviation of the stock's rate of return by the ISD was initially suggested by Latane and Rendleman (1976). They derive standard deviations of continuous price relative returns which are implied in actual call option prices on the assumption that investors behave as if they price options according to the Black-Scholes model. The standard deviation implied by any given option price can be found by using numerical search to have a close approximation. The procedure is to equate the right-hand side of the Black-Scholes equation within $\pm$ \$0.001 of the actual call price for a sample of observed call option prices.

Latane and Rendleman's data set contain weekly closing option and stock prices on twenty-four firms during the period October 5, 1973 to June 28, 1974. The weekly hedge returns are calculated separately for over- and under-valued options under various criteria in selecting options and determining the hedge-ratios. The criteria are the individual option's ISD the underlying stock's WISD, and the ex-post time-series standard deviation. The strategy, based upon the historical series of rates of return, is considered to be naive strategy against which the returns generated from the use of the WISDs. Under the assumption that the WISD is the proper measure of the standard deviation, Latane and Rendleman expect absolute higher returns for strategies employing WISDs. They find all the portfolios employing WISDs to produce significant (at 5 percent) mean excess returns, which are also consistently higher than those using the ex-post standard deviations. Latane and Rendleman conclude that "the WISD based upon the Black-Scholes model is useful, not only in determining proper hedge position, but also in identifying relative over- and undervalued options" (p. 375).

From comparing the WISDs to the time series standard deviations they conclude that options were generally overpriced in terms of the Black-Scholes model during the sample period. While the WISDs for a given stock are not perfectly stable, they find a strong tendency for the cross-section estimators to move together over time. This led Latane and Rendleman to conclude that while the model can be used effectively to determine whether individual options are properly priced, "the model may not capture the process determining option prices in the actual market" (p. 375).

Schmalensee and Trippi (1978) assume the volatility of the Black-Scholes model and impute the standard deviation from weekly observations over the period April 1974 to May 1975, for six weekly traded stocks and their options. Based on previous studies that claimed that the Black-Scholes model worked poorly for deep-in-the-money and deep-out-of-the-money options, they eliminate these options from their sample. The purpose of their study is to investigate "the determinant of changes over time in the market's collective expectations of common stock volatility" (p. 130).

Schmalensee and Trippi use an arithmetic average of ISDs, based on closing prices, as an estimator of the standard deviation and check the behavior of the changes in the average over time. They find the changes in volatility to contain nonwhite-noise elements, which would indicate market inefficiency, given the validity of the Black-Scholes model. But, because actual volatilities change over time, as do the average ISDs, the Black-Scholes model may be inappropriate.

Similar to the results of Latane and Rendleman, they also find some market-based effects that influenced the change in volatility across stocks. Surprisingly, a weak relationship is found between changes in the average ISDs and the ex-post time-series standard deviations. Also surprising is the strong impact of the direction of change of stock prices on the changes in the ISDs. These results are not consistent with the Black-Scholes model framework, nevertheless, the Black-Scholes model can be useful in predicting such ex-ante changes.

Beckers (1980) notes that there is a basic inconsistency in using the model to obtain predictions of a presumably nonstationary variance. However, the results of Latane and Rendleman and Schmalensee and Trippi indicate that the Black-Scholes model is still valuable in predicting future volatilities. It may be the case that the model is not very sensitive to violations of the nonstationary assumption. Beckers extends the above studies by considering alternative weighting schemes for the volatility estimator and by using a dividend-adjusted model. For a sample of CBOE options during the period 13 October 1975 to 23 January 1976, Beckers finds the ISD derived from an at-the-money option to be a better predictor of the actual time-series standard deviation over the life of the option than a weighted ISD. He also finds the ISDs to be extremely volatile over time and suggests using an intertemporal arithmetic average. While the instability of the ISD may indicate market inefficiency, Beckers attributed it to the trading mechanism and especially to the lack of market synchronization. Expanding the sample over the period May 1975 to July 1977 and using five-day arithmetic averages of ISDs, Beckers reconfirms his earlier findings. In the regression analysis he finds that past time-series standard deviations add to the explanation of the ex-ante time series standard deviation above and beyond what is explained by the ISD. This, again, may indicate market inefficiency in using available data; it may also indicate model misspecification or data problems. Some support to the latter is received from analyzing a sample of transaction data that show that closing price may seriously distort the ISD calculations.

While all the estimated ISDs are point estimation usually based on closing prices for shares and options, little is known about their distribution properties. Brenner and Galai (1981) examine some properties of the ISDs based on transactions data on five stocks for ninety-eight trading days starting from 3 June 1977 to 21 October 1977. It appears that there are significant deviations of the ISDs based on the last transaction for the day (LISD) from the daily average ISD (AISD). Longer maturity options exhibited a tendency to higher AISDs than short maturities, which implies that options with a long life were overpriced relative to short maturity options. Moreover, the distributions of AISDs over time were not stationary. There were also significant differences of AISDs across striking prices. Thus, the results are consistent with findings of other researchers, and lead to the rejection of the joint hypotheses that the Black-Scholes model is valid-that the stocks and options markets are efficient and synchronous, and that the estimation procedure are correct.

Black-Scholes' most exacting assumption disallows income distributions on the underlying security. However, less than five percent of the options listed on the CBOE are written on non-dividend-paying stocks. A simple approximation for the value of the American call is the value of a European call, F(S,T,K), where S is the stock price net of the present value of the escrewed dividend payment[6], $S_\tau = P_\tau - \alpha D e^{-r(t-\tau)}$ for $\tau < t$, and $S_\tau = P_\tau$ for $t \le \tau \le T$. This approximation ignores a second dividend-induced effect in that it presumes that the call will not be exercise prior to expiration. To compensate for this possibility Black (1975) suggests an approximate value equal to the higher of the values of a European call where the stock price net of the present value of the escrewed dividend is substituted for the stock price and a European call where the time to ex-dividend is substituted for the time to expiration, that is

$$\max[C(S,T,K), C(P,t,K)]$$

The probability of early exercise is assumed to be zero and one for the first and second option, respectively, in the maximum operator.

Whaley (1982) examines the pricing performance of the three methods for valuing American call options on dividend-paying stocks. The methods include: (1) the simple approximation obtained by substituting the stock price net of the present value of the escrewed dividend into the Black-Scholes formula, (2) the Black approximation obtained by taking the higher of the Black-Scholes formula value using stock price net of the present value of the escrewed dividend and the Black-Scholes formula value using the time to ex-dividend as the time to expiration variable, and (3) the correctly specified equation for the American call option on a stock with a known dividend.

The data employed in his study consist of weekly closing price observations for all CBOE call options written on 91 dividend-paying stocks during the 160 week period January 17, 1975 through February 3, 1978. The prices of stocks, options and Treasury Bills were

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[6] Assuming that at the ex-dividend instant, t ($t < T$), the stock pays a certain dividend, D, and the stock simultaneously falls by a known amount, α D.

compiled from the Wall Street Journal. According to the American call option model specification, the option's underlying stock must have exactly one dividend paid during the life of that option. Thus, options with expiration dates before the stock's next ex-dividend date and options with time to expiration including more than one ex-dividend date were eliminated.

Unlike the previous empirical studies, which compute a single volatility estimate on the basis of all the options written on a stock at a particular point in time, Whaley uses only those options which share a common maturity. Patell and Wolfson (1979) suggest and demonstrate empirically that the standard deviation implied by the price of a longer-lived option written on a stock is greater than the standard deviation implied by the price of a shorter-lived option if there is an anticipated information event between the expirations of the two options. Eventhough, previous researchers employ weighted implied standard deviations as predictors for future return volatilities, the estimates of standard deviations in his study are determined by minimizing the sum of squared deviations of the observed call prices from the model's prices. This criterion allows call prices to provide an implicit weighting scheme that yield an estimate of standard deviation which has as little prediction error as is possible.

Comparing the relative prediction error of the American call option to the other simple approximation methods, Whaley finds that it is not only markedly lower than those of the alternative models, but also not systematically related to : (1) the degree to which the option is in-the-money or out-of-the-money, (2) the probability of early exercise, (3) the option's remaining time to expiration, and (4) the stock's dividend yield. However, the American call formula does not eliminate, although it reduces, the association between prediction error and the standard deviation of stock return. All models tend to overprice options on high-risk stocks and to underprice options on low-risk stocks. This phenomenon is consistent with both Black and Scholes and MacBeth and Merville's findings.

Barone-Adesi and Whaley (1986) estimate the value of the ex-dividend coefficient α implicitly by allowing observed call option transaction prices to provide the best estimate of the relative stock price decline. To estimate the parameters, α and, σ, call option transaction prices are regressed on the Roll model prices by using maximum likelihood procedure. The data used in their study consist of transaction information for all CBOE call options traded during the first three months of the calendar years 1978 and 1979. Their test results indicate that the expected relative stock price decline is not meaningfully different from one.

In general, the estimates of option prices using variance estimates are biased because the Black-Scholes formula is non-linear in the variance, and unbiasedness is not preserved under a non-linear transformation. The bias can be reduced by selecting a larger sample size in order to lower the variance of the variance estimate. However, there is growing empirical evidence, for example, Christie (1982), and Schwert (1989), that the variance is not constant over time. As the result, the longer is the time series of observations used to estimate the variance rate, the more biased the variance estimate may be.

Because of the non-linearity of the Black-Scholes formula, it is not possible to insert the estimated variance in the formula to get unbiased estimates of the option price. The non-linearity exists from the standard normal cumulative distribution function. Butler and Schachter (1986) circumvent this problem by using Taylor series expansion to express the standard normal cumulative distribution function around zero to arbitrary accuracy. Furthermore, they derive the unbiased estimator of the series of the odd powers of the standard deviation and then substitute them into the expression of the standard normal cumulative distribution function.

Finally, Butler and Schachter compare the performance of the unbiased estimator and the Black-Scholes option price using an unbiased variance estimate. Basing on the simulations, the unbiased estimator performs better on the whole. The bias in the usual estimator is many orders of magnitude greater than the unbiased estimator. On average, for a contract to purchase 100 shares of a stock with a price of $50, the bias is on the order of $0.33 for the usual estimator whereas the maximum bias is less than 10^{-8} for the unbiased estimator. The absolute bias is greatest for at-the-money options, with an average bias of $0.67 on a similar contract.

However, the biased estimator does not solve all problems related to the determination of Black-Scholes option prices. In order to compare the performance of the Black-Scholes price and the unbiased estimator of the Black-Scholes model price, an empirical test is necessary. Furthermore, the unbiased estimator does not address the problem with non-stationary in the variance, model misspecification and dividend effects.

4.4 Direct Comparisons

The first paper that will be introduced in this section is a descriptive analysis of how market prices of call options compare to prices predicted by the Black-Scholes option pricing model. Actually, Black (1975) has stated that market prices of call options tend to differ in certain systematic ways from the values given by the Black-Scholes model for options with less than three months to expiration and for options that are either deep in or deep out of the money. Based on the previous statement , MacBeth and Merville (1979) inspect market and Black-Scholes call prices under the assumption that the Black-Scholes model correctly prices at the money options with at least ninety days to expiration.

The basic method of their analysis is to numerically solve the Black-Scholes option prices for the unobservable quantity, the variance rate, σ^2. The values of σ^2, implied variance rates, can be found by substituting the observed market price into the Black-Scholes formula and numerically solve for the
implied variance rates.

Their sample consists of daily closing prices of all call options traded on the Chicago Board of Trade Options Exchange for American Telephone and Telegraph (ATT), Avon products (AVON), Eastman Kodak (ETKD), Exxon, International Business Machines (IBM), and Xerox from December 31, 1975 to December 31, 1976. Option prices and prices of the underlying stocks were taken from the Wall Street Journal. The riskless return was imputed from bid and ask yields reported in the Wall Street Journal for United States Treasury Bills by selecting a Treasury Bill that expired just beyond the expiration date of the option and then followed the yield on that Bill through time. Dividend information came from Standard and Poors Stock Record. They have examined their data for the period between ninety and one hundred days prior to expiration and find no evidence of an early exercise effect.

In their study, MacBeth and Merville found that on any given day different market prices of options written on the same underlying stock yield different values of the implied variance rates and these implied variance rates for the same option changed through time. Furthermore, their empirical results show that the implied variance rates decline as the exercise price increases.

As mentioned earlier that analysis is based on the assumption that the Black-Scholes model correctly prices at the money options with at least ninety days to expiration. It follows from this assumption that the at the money implied variance rate is the proper or 'true' variance rate. From this stand point, the Black-Scholes model must yield call option prices which exceed observed market prices for option out of the money and call option prices which are less than the observed market prices for options in the money because the implied values of σ decline as the exercise price increase.

In order to investigate the behavior of the Black-Scholes model prices and observed market prices, the implied variance rate for at the money option needs to be determined. MacBeth and Merville estimated the implied value of σ by regressing the implied σ for option j on the security i on day t, σ_{ijt}, on the variable representing the degree of which the option is in or at or out of the money, m_{ijt}. That is, they estimate the following regression model each day,

$$\sigma_{ijt} = \theta_{iot} + \theta_{i1t} m_{ijt} + \varepsilon_{ijt}$$

where m_{ijt} is defined as

$$m_{ijt} = \frac{S_{it} - K_{it} e^{-rt}}{K_{ijt} e^{-rt}}$$

The estimated values of θ_{iot} , $\hat{\theta}_{iot}$ is their estimate of the σ that would be implied by the Black-Scholes model on day t for an at the money option written on security i. They already excluded options with less than ninety days to expiration from the calculation of $\hat{\theta}_{iot}$.

To compare the observed market prices and the Black-Scholes model prices they relate the difference between C_{ijt}, the market price on day t of option j on security i with the exercise price K_{ij} , and the Black-Scholes model price of the same option, $C^m(\hat{\theta}_{iot})$, to the extent to which the option is in or out of the money as measured by m_{ijt} .

Initially, they relate m_{ijt} to v_{ijt}, the difference between the market price of an option and the Black-Scholes model price expressed as a percentage of the Black-Scholes model price.

$$v_{ijt} = \frac{C_{ijt} - C^m(\hat{\theta}_{iot})}{C^m(\hat{\theta}_{iot})}$$

The relationship of m_{ijt} and v_{ijt} is expressed on scatter diagrams which confirm the impressions gained from inspecting the regression results of C_{ijt} against the degree the option is in-the-money, m_{ijt}, and against the time to maturity.

The authors conclude that if the Black-Scholes model correctly prices at-the-money options with medium to far maturities (such as, with at least ninety days to maturity), then the Black-Scholes model underestimates (overestimates) market prices for in-the-money (out-of-the-money) options. The extent of the mispricing decreases as the time to expiration decreased.

In their second paper (1980), they confront the Black-Scholes model with the Cox model of constant elasticity of variance, assuming that the latter should reduce the mispricing of Black-Scholes prices observed in the first paper.

MacBeth and Merville (1980) have maintained all of the assumptions of the Black-Scholes option valuation model, except that the stochastic stock price process is generalized to a constant elasticity of variance process. The family of constant elasticity of variance diffusion process can be described by the stochastic differential equation as,

$$dS = \mu S dt + \delta S^{\theta/2} dz$$

where dz is a Wiener process, and $\theta > 0$. The instantaneous variance of the stock price is $\delta^2 S^\theta$ and the elasticity of this variance with respect to the stock price equals θ. The instantaneous variance of the percentage price change or return, σ^2, is given by the equation

$$\sigma^2 = \delta^2 S^{(\theta-2)}$$

which is a decreasing function of the stock price for $\theta < 2$. When θ equals two, the instantaneous variance of return is a constant, θ^2, and the stochastic process generating returns is a lognormal diffusion process, the process assumed in the Black-Scholes valuation model.

The authors repeat the 1979 tests and conclude that "the stochastic-process-generating stock prices can best be considered as a constant elasticity of variance process" (1980, p. 299). In his discussion of the MacBeth and Merville paper (1980), Manaster (1980) points out the empirical superiority of the Cox model over the Black-Scholes model is not surprising at all. "Given that the Cox model includes Black-Scholes model as a special case, it is clear that the Cox model must explain observed option prices at least as well as the Black-Scholes model" (p. 301).

Thorp and Gelbaum (1980) also believe, for empirical and theoretical considerations, that the constant elasticity of variance is a better model than the Black-Scholes. But, contrary to the findings of MacBeth and Merville, Thorp and Gelbaum claim that their experience in trading CBOE options while applying the Black-Scholes model is that this model tends to underprice the out-of-the-money options. The same phenomenon is also stated by Black (1975). The differences in results between the two studies may stem from changes in the average volatility of the market; better adjustment for the potential early exercise of the CBOE options in the latter study; and/or changes in the procedure for estimating the variance of the rate of return. MacBeth and Merville use the average implied standard deviation for at-the-money options and hence their prices are relative to the average (assuming that at-the-money options are efficiently priced). Thorp and Gelbaum use the historical stock volatilities.

Many studies have shown the biasness of the Black-Scholes model prices in predicting market prices of options. This bias may be due, in part, to violation of the Black-Scholes assumptions of European option contracts with no dividend distributions on the underlying stock prior to expiration of the contract.

Blomeyer and Klemkosky (1982) study the performance of the Black-Scholes model prices compare to the Roll option-pricing model. The objective of their paper is to examine the ability of each model to identify overvalued and undervalued option contracts on an ex-post basis by using the hedging technique suggested by Black-Scholes (1972).

Blomeyer and Klemkosky use the MacBeth and Merville technique of plotting the relative deviation of the model price from the market price against the degree the option is in-the-money to compare the Black-Scholes model to the adjusted Roll model suggested by Whaley (1981). The Roll model, which gives the price of the dividend-unprotected American call option, is more general than the Black-Scholes model and hence is expected to give more accurate predictions of market prices. The use of the Black-Scholes model prices for the stock paying a cash-dividend distribution prior to the option expiration date will bias the model prices downwards due to the possibility of early exercise of the option to capture the dividend payment.

Transaction data for CBOE options written on eighteen stocks, for one trading day per month from July 1977 to June 1978, are employed in their research. The weighted implied standard deviation, WISD, given by Chiras and Manaster (1978) is used to provide stock-return variance estimates for both the Black-Scholes and Roll models. Their methodology is to compare the means excess return on hedges of the two models aggregated across all transactions on all options written on a particular stock. The expected hedges excess return will be positive if the model is successful at identifying overvalued and undervalued call options and the hedge is established properly. The superior model should consistently develop higher holding-period returns.

Based on 9108 observations of option prices they drew the conclusion that the mean-holding-period returns realized from the Roll model exceeded those realized from the Black-Scholes model for seven of the eighteen stocks, with none of the differences between the mean-holding-period returns for the two model significant at the .05 level. Furthermore, fifteen of the mean-holding-period returns were significantly different from zero at .05 level and positive for both models. The graphical relationship between the relative deviation of the model price from the market price and the degree the option is in-the-money for both models shows that both models have virtually identical pricing-bias characteristics. The results are contrary to the MacBeth and Merville (1979) findings, but consistent with Thorp and Gelbaum (1980); both models undervalue, relative to the market, the out-of-the-money options and price fairly well the at- and in-the-money options.

A recent study by Choi and Shastri (1989), examines the relationship between bid-ask spreads and estimates of volatility based on observed sequences of transaction prices. They derive some implications of this relationship for the use of the Black-Scholes model in the pricing of options. They show theoretically and empirically that volatility estimates based on observed sequences of transaction prices overestimate the 'true' volatility of returns on a security, with the magnitude of the overestimation being an increasing function of the level of the volatility. If market prices are determined by the Black-Scholes model based on 'true' variance but model values are obtained using observed volatility, then the model will overestimate the high variance stocks.

The sample used in their study consists of all reported quotations for call options traded on the CBOE for four randomly selected dates, December 6, 1977; February 28, 1978; June 16, 1978 and August 31, 1978. In order to avoid problems associated with time to maturity and moneyness biases, only short-term maturity and near-the-money options are used in their empirical tests. In addition, they eliminate options whose underlying stocks are expected to pay dividends over the life of the options because the Black-Scholes model valid only for European calls.

According to Choi and Shastri's framework, the observed return on a security ($\bar{r}_t$) consists of two components, the 'true' return (r_t), and the spread induced return (r_t^s) which can be described by

$$\hat{r}_t = r_t + r_t^s$$

With a specific functional form[7] of the spread induced return on a security (r_t^s), the covariance between the 'true' return on an asset (r_t) and the spread induced return is zero. The solution to the variance of the spread induced return is

$$\sigma_s^2 = (1-\rho)s^2 , \quad \text{and} \quad \hat{\sigma}^2 = \sigma^2 + \sigma_s^2$$

where s is the percentage bid-ask spread on a security, and ρ is the conditional probability of a transaction at the bid (ask) price following one at bid (ask) price.

This relationship implies that the spread-induced variance is an increasing function of the percentage bid-ask spread on the stock whereas it is a decreasing function of the conditional probability of successive transactions being the same type.

In their empirical tests, they assume that ρ is 0.5. To test the hypothesis that the magnitude of the bid-ask spread is an increasing function of the variance of returns, they regress s on σ^2, which yields the following result

$$s = 0.0027 + 7.25\sigma^2$$

The t-statistic of the slope coefficient is 9.38, far enough to support their hypothesis. They also test whether the differences between the Black-Scholes model prices based on observed variances and market prices of options are an increasing function of the volatility of the underlying security by the following regression result.

$$C^m - C^s = -0.43 + 13.96\hat{\sigma}$$

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[7] $r_t^s = s I_t$ where

$$I_t = \begin{cases} +1 \text{ if transaction at t is at ask and } t-1 \text{ is at bid} \\ -1 \text{ if transaction at t is at bid and } t-1 \text{ is at ask} \\ 0 \text{ otherwise} \end{cases}$$

The t-statistics of the slope coefficient is significantly greater than zero at the 5% significance level which indicates the expected variance bias. Thus, using the volatility estimates based on observed sequences of security prices will overestimate the market prices with the size of the bias is an increasing function of the volatility of the underlying security. This variance induced bias is caused by the inclusion of the spread induced variance component, $\tilde{\sigma}^2 = \sigma^2 + \sigma_s^2$. Thus, to reduce this bias, one should use the 'true' variances instead of the observed stock volatilities. To see this relationship, Choi and Shastri regressed $C^m - C^a$ on the true variance, $\sigma^2 = \tilde{\sigma}^2 - \sigma_s^2$, to yield the following estimated equation,

$$C^m - C^a = -0.41 + 10.85\sigma^2$$

Although, the use of the variance adjusted for spread volatilities reduces the bias, the magnitude of the slope coefficient indicates that options on high volatility stocks are still overpriced relative to those on low volatility. This means that the spread induced volatility is not of sufficient magnitude to explain the variance bias exhibited by the Black-Scholes model.

4.5 Simulation Results

In general, the assumptions of the Black-Scholes model do not hold perfectly in reality. The question, therefore, is how robust the model is and how sensitive the results will be where deviations from the basic assumptions are observed.

Boyle and Ananthanarayanan (1977) examine the implications of using an estimate of the variance in option-valuation models. They show the bias in the average option value to be rather small, even for relatively small samples of a stock' rates of return. However, the dispersion of the distribution of option prices may be quite significant.

Boyle and Emanuel (1980) analyzed the distribution of the returns on the hedge portfolio when rebalancing takes place at discrete points in time. Their studies were based on a world where the assumptions of the Black-Scholes model held with certainty. The discrete return on the hedge (HR) allowing for the opportunity cost of its equity is

$$HR \quad = \quad \Delta C - \Delta S\, N(d_1) - r\,\Delta t\left[C - SN(d_1)\right] + o(\Delta t)$$

Expanding the option-pricing function by using Taylor's theorem yields

$$\Delta C \quad = \quad C_s \Delta S + C_t \Delta t + \frac{1}{2} C_{ss} (\Delta S)^2 + \frac{1}{2} C_{tt} (\Delta t)^2 + C_{st} (\Delta S \Delta t) + \theta$$

where θ contains terms of $\Delta S^i \Delta t^j$ with $i + j > 2$, and $o(\Delta t)$. Substituting the stock's price dynamic; that is, $\frac{\Delta S}{S} = \alpha \Delta t + \sigma u \sqrt{\Delta t} + o(\Delta t)$; into the above equation produces,

$$\Delta C \quad = \quad C_s \Delta S + C_t \Delta t + \frac{1}{2} C_{ss} \sigma^2 S^2 u^2 \Delta t + o(\Delta t)$$

Substituting the values of C_t , C_{ss} and rearranging terms, HR can be express[8] as

$$HR \quad = \quad \frac{\sigma S}{2\sqrt{T}} Z(d_1)(u^2 - 1)\Delta t$$

The higher power terms of Δt were neglected, and $Z(.)$ is the density function of standard normal distribution. Boyle and Emanuel define,

$$\lambda \quad = \quad \frac{\sigma S}{2\sqrt{T}}$$

$$y \quad = \quad u^2 - 1$$

Thus, HR becomes

$$HR \quad = \quad \lambda\, y\, \Delta t$$

As the results, the return on the hedge portfolio can be expressed as the product of three components.

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[8] F_x, and F_{xy} are partial derivative of F with respect to x, and second order partial derivative of F with respect to x, and y, respectively; For more details, see Boyle and Emanuel (1980).

- The deterministic function of the variables S, T, K, r, and σ^2 which are contained in λ evaluated at the time when the hedge is constructed.
- The stochastic component of HR, y, which is drawn from a chi-squared distribution having zero mean.
- The time interval between adjustments to the hedge.

The value of HR will be negative if and only if $|u| < 1$. As was mentioned by Boyle and Emanuel, the drawing of u is from the unit normal distribution which in turn HR will be negative about 69 % of the time. On another hand, for relative large stock price movement during the interval Δt, $|u| > 1$, the hedge will only yield positive returns on the relatively few occasions. The skewness of the y-distribution makes the t-statistics for confidence interval of the hedge portfolio returns biased.

The Black-Scholes assumptions are assumed to hold perfectly in their simulations. They performed ten thousand simulations of hedge performance over ten, twenty, forty, and eighty trading intervals. The specific values of parameters used in the simulation are ; r = 0.008/month, σ = 0.01/month, S/K = 0.9, 1, 1.1, and dt = 0.05 months which is one trading day. The first percentiles for the t-statistics are -5.6, -4.4, -3.7, and -3.5, for ten, twenty, forty and eighty trading intervals respectively. The ninety-ninth percentiles are 1.7, 1.7, 1.8, and 1.9 respectively. For small sample sizes, the t-statistics will be biased downwards. This is because a negative mean return is caused by a predominance of negative daily hedge returns. The negative returns are closely bunched together and the variance estimate tends therefore to be low. This leads to a large negative t-statistic. Conversely, a positive mean return is caused by a predominance of positive widely dispersed daily returns. The t-statistic tends to appear insignificant because of the resulting high variance estimate. Another study that uses simulation to check the robustness of the Black-Scholes model is by Bhattacharya (1980). To eliminate the problem of measurement errors, he uses model prices for options. These values coupled with the actual distribution of stock prices, are used in creating neutral hedge positions.

Bhattacharya created hypothetical options with maturities of 1 through 5 days, 2 and 3 weeks, 1, 1.5, 2, 3, 4, 6, and 9 months. For each maturity of one month or longer, four exercise prices to prevailing stock price ratios were employed. Of these, K/S ratios of 1.25 and 1.125 represent deep-out-of-the-money, 1.0 and 0.875 represent at-the-money, and deep-in-the-money, respectively. For very short maturities, in view of the rapidly changing $\partial C / \partial S$ for near-the-money short maturity options, two more K/S ratios (1.05 and 0.95) were added to obtain finer grid.

Observed stock returns for the 18-month period ending October 21, 1977, resulted in 378 data points for each of the 91 stocks which had options traded on them on the CBOE and for which daily returns were available for each trading day from the CRSP daily returns data. In order to exclude the effect of stock splits or stock dividends, he excluded the stocks which had stock dividends or stock splits over the 18-month period.

If the mathematical structure of the formula is correct, according to Bhattacharya, the average hedge return will be insignificantly different from zero. Significant excess hedge returns may be attributed to the deviation of the actual distribution of stock prices from the assumed stationary, lognormally distribution, and "would be considered as evidence in support of the hypothesis that when observed stock prices are used as input, the Black-Scholes formula exhibits pricing biased" (p. 1085).

The major result of the study is that options near-in-the-money and near-out-of-the-money with maturities of five days or less are frequently statistically significant whereas at-the-money one-day-to maturity options provide statistically and operationally significant excess returns.

CHAPTER 5. MISPRICING OF THE BLACK-SCHOLES MODEL

The literature, which includes empirical studies, confirms that the Black-Scholes formula tends to exhibit systematic empirical biases. The bias patterns often occur with respect to exercise price, time-to-maturity, and stock volatility. Specifically, the model tends to underprice options near maturity or on low-variance stocks and to overprice options on high-variance stocks. Additionally, the exercise price bias is such that models using pre-1977 data underprice out-of-the-money options and overprice in-the-money options. The patterns are reversed in models using post-1977 data.

When model price is compared with observed option price, stock volatility needs to be estimated. One candidate used widely as a proxy for expected volatility is implied volatility, as suggested first by Latane and Rendleman (1976). This estimate, however, is sensitive to change in option positions. Day and Lewis (1988) explain that there are two significant sources of noise that makes the implied volatilities of options sharing time-to-maturity and exercise price unequal. These sources are the uncertainty of whether option and stock prices reflect bid or ask levels and the failure to observe simultaneously option price and underlying security price.

Rubinstein (1985) investigates these relations and reports that out-of-the-money options with shorter lives have greater implied standard deviations (ISDs), which, in turn, implies that such options are relatively overpriced. He confirms MacBeth and Merville's (1979) findings that ISDs tend to increase with decreasing striking prices for 1976 data and that this biased pattern continues until approximately October 1977. Towards the end of 1977 and during 1978, however, the bias is reversed. For at-the-money options during the first subperiod, the longer the option life, the greater the relative ISD. The reverse is observed during the second subperiod.

The empirical evidence of previous studies indicates that the implied volatilities not only contain information on the anticipated change in stock returns but also *relate systematically to the option position with respect to time-to-maturity and degree-of-moneyness.* Thus, using only a single estimated implied volatility to describe the whole series of option data may distort the true behavior of market option prices. On the other hand, if the estimated implied volatility of the out-of (at or in) -the-money is used to construct the model price for the out-of (at or in) -the-money options, the Black-Scholes model price may be related more closely to the observed out-of (at or in) -the-money market option prices.

Our analysis is basically similar to that of MacBeth and Merville (1979). But to estimate the implied volatility, our study uses a nonlinear least squares estimation procedure. Such a technique is suggested by Whaley (1982), Day and Lewis (1988), and Stephan and Whaley (1990). Unlike previous empirical studies, which compute only a single volatility estimate for all options written on a stock, our study uses four different approaches to estimate implied volatilities. These are 1) computing a single estimated implied volatility from all options, 2) separating options into five groups based on their degree-of-moneyness and computing the corresponding implied volatilities, 3) using options sharing time-to-maturity so as to estimate implied volatilities[1] and 4) estimating implied volatilities for options sharing time-to-maturity and degree-of-moneyness. These four approaches permit testing of the stationarity of implied volatility across maturities and/or striking prices. Although disagreeing strongly with the Black-Scholes assumption of constant volatility, our findings support the *visual* examinations of Latane and Rendleman (1976), Schmalensee and Trippi (1978), MacBeth and Merville (1979), and Beckers (1980) and the nonparametric test

[1]Whaley (1982) and Stephan and Whaley (1990) apply this technique.

of Rubinstein (1985) in that there is a fundamental inconsistency in using the Black-Scholes model to obtain predictions of a presumably nonstationary variance. Nevertheless, the Black-Scholes model remains of value in predicting of market option price.

Investingating the mispricing of the Black-Scholes model, MacBeth and Merville (1979), Whaley (1982), Gultekin, Rogalski, and Tinic (1982), and Choi and Shatri (1989) regress the difference between observed and model option prices[2] on option maturities and/or degree-of-moneyness. There is a "sizable positive autocorrelation in the error terms..., which, in turn, probably results from the effects of one or more explanatory variables left out of the regression model" (MacBeth and Merville, 1979, p 1185). Thus, "the statistics of the estimated coefficients... cannot be strictly interpreted because they overstate the statistical significance of the estimated coefficients" (MacBeth and Merville, 1979, p 1184).

To solve the statistical problems faced by previous studies, we construct a robust model to test the systematic over- or underpricing of the Black-Scholes model by incorporating the effects of striking price, time-to-maturity, and autoregressive error directly into model price. This technique improves the efficiency of estimated model parameters.

5.1 Past Empirical Studies

In a seminal paper, Black-Scholes (1972) presented a closed-form valuation model for European options. Merton (1976) demonstrated later that the Black-Scholes model for valuing European call options is applicable equally to American call options written on stocks not paying dividends and on call options protected against dividend payouts. Closed-form solutions to option prices for stocks paying dividends at known, discrete time intervals have been established by Roll (1977) and by Geske (1979).

[2]In testing the systematic variance biases of the Black-scholes model prices, Choi and Shatri regress the prediction error on the 'observed' volatility computed from daily returns of CRSP Daily Returns files.

Despite the enormous interest that the Black-Scholes model generated in the academic and the professional communities, the model incorporates only a few empirical tests. An initial test by Black and Scholes (1973) used a sample of 2,039 call and 3,052 straddle contracts from diaries of an option broker from 1966-1969. The researchers found systematic differences between the actual option premiums observed in the market and those estimated by the model. In particular, actual premiums of options written on high-variance stocks were lower than premiums predicted by the model. On the other hand, premiums of options on low-variance stocks were higher than those predicted by the model. Black-Scholes attributed this bias to their inability to obtain accurate estimates of the variance rate on common stocks.

In their initial paper, MacBeth and Merville (1979) described the standardized difference between the actual and the Black-Scholes model call prices as

$$\frac{C_t^A - C_t^M}{C_t^M} \quad ,$$

where C_t^A and C_t^M are the actual and the Black-Scholes model call prices, respectively. For a sample of daily closing prices for six stocks from 31 December 1975 to 31 December 1976, MacBeth and Merville observed this statistic to be an increasing function of the extent to which the option is in- or out-of-the-money:

$$\frac{S_t - K_t e^{-rt}}{K_t e^{-rt}} \quad ,$$

For deep-in-the-money options, the difference between observed market prices and model prices seemed constant and positive. For deep-out-of-the-money options, the difference was negative. Regression results of $C^A - C^M$ against the degree to which the option is in-the-money and against the time-to-maturity, confirm the impressions gained from inspection of individual scatter diagrams.

The authors concluded that if the Black-Scholes model correctly prices at-the-money options with medium to far maturities, i.e., options with at least 90 days to maturity, then the model underestimates (overestimates) market prices for in-the-money (out-of-the-money) options. Extent of mispricing decreases with time-to-expiration.

MacBeth and Merville assume market efficiency and attribute deviations of model price from actual price to model weakness, especially the assumption of a constant variance in rate of return. In their second paper (1980), they confront the Black-Scholes model with the Cox model of constant elasticity of variance, assuming that the latter should reduce the mispricing of the Black-Scholes model observed in their own first paper. They repeat the 1979 tests and conclude that "the stochastic-process-generating stock prices can be best considered... a constant elasticity of variance process" (1980, p. 299).

In his discussion of the MacBeth and Merville paper (1980), Manaster (1980) points out that the empirical superiority of the Cox model to the Black-Scholes model is not at all surprising: "Given that the Cox model includes the Black-Scholes model as a special case, it is clear that the Cox model must explain observed option prices at least as well as the Black-Scholes model" does (p. 301). He suggests that an ex-ante trading strategy be performed to determine which model is superior.

Comparing the Black-Scholes model with the Roll (1977) model, Blomeyer and Klemkosky (1982) use the MacBeth and Merville techinque of plotting the relative deviation of model price from market price against the degree to which the option is in-the-money. The Roll model, which gives the price of the dividend-unprotected American call option, is more general than the Black-Scholes model and hence should yield more accurate predictions of market prices. Transaction data for CBOE options written on 18 stocks for one trading day per month from July 1977 to June 1978 are used. Blomeyer and Klemkosky draw the conclusion based on 9,108 observations of option price, that the two pricing models have virtually identical pricing-bias characteristics. The results are contrary to those of MacBeth and Merville (1979), however.

Instead of using a single implied volatility for all maturities, Whaley (1982) uses the Black-Scholes model to calculate the maturity-specific implied stock volatilities. In comparing the American call option valuation with the Black-Scholes formula and the Black approximation formula, he finds that all models demonstrate a slight tendency to overprice low-priced options and to underprice high-priced ones. For options whose prices exceeded $0.50 and whose remaining lives included exactly one ex-dividend date, the relative prediction error of the American call is markedly smaller than that of the alternative models and is systematically unrelated to the degree to which the option is in-the-money or out-of-the-money, the probability of early exercise, the option's remaining time-to-expiration, or the dividend.

Gultekin, Rogaski, and Tinic (1982) present empirical evidence to explain the accuracy of the Black-Scholes model in estimating premiums of options approaching expiration. Option values obtained from the Black-Scholes model, on average, overestimate prices of options written on high-variance stocks and on in-the-money options but underestimate prices of options written on low-variance stocks and out-of-the-money options. Beyond that, the differences between values calculated by the model and actual option prices increase with time to expiration. That is, the shorter the maturity, the more accurately the Black-Scholes model estimates actual prices.

The empirical study of Levy and Byun (1987) attempts to test the systematic bias of the Black-Scholes model by using the confidence interval of the estimated variance to derive the confidence interval of model call-option value. Even when the variance confidence interval is considered, a systematic deviation between the theoretical range of the option price values and the observed market price still exists. The researchers state that "if the stock variance is constant over time, the interpretation of the Black-Scholes model is wrong" (Levy and Byun, 1987, p. 355). If, however, stock variance changes over time, they state that "the implied volatility in options market prices had a tendency to be significantly higher than the estimate that could have been obtained from historical data" (Levy and Byun (1987), p. 355).

5.2 Methodology

Two main issues will be discussed in this section. The first concerns testing the implications of the Black-Scholes model assumption of constant stock volatility; the other, developing a robust model to reinvestigate the systematic over- or underpricing of the model. Basically, both estimated stock volatility and mispricing are related to changes in option position. Thus, it becomes useful to describe how options are classified with respect to both in-the-money degree and maturity.

In general, options sharing a maturity have a closed estimate of implied volatility and the same mispricing behavior. Our study, therefore, categorizes options according to expiration month, as indexed by τ. Because the option data used in this thesis are from March 1990, τ can assume values from 1 to 5. For example, the expiration months of GE option contracts sold in March 1990 can be in March, April, May, June, or July, which correspond with the values $\tau = 1, 2, 3, 4$, and 5, respectively.

The estimated implied volatility and the mispricing degree also are related closely to the moneyness degree. On the basis of stock (S)-to-striking (K) price ratio, most empirical studies partition options into three groups, viz., out-of-the-money ($S/K \leq .9$), at-the-money ($.9 < S/K < 1.1$), and in-the-money ($S/K \geq 1.1$). But the option data used in our study contain a great percentage of at-the-money options (greater than 85%). To refine the analysis and to examine the at-the-money option behavior, the study classifies these options into three categories, viz., at-the-money near out-of-the-money ($0.9 < S/K \leq 0.975$), at-the-money ($0.975 < S/K \leq 1.025$), and at-the-money near in-the-money ($1.025 < S/K < 1.1$). Thus, representing moneyness degree, k ranges from 1 to 5, as specified:

$$k = \begin{cases} 1 \text{ if } S/K \leq 0.9, & \text{out-of-the-money} \\ 2 \text{ if } 0.9 < S/K \leq 0.975, & \text{at-the-money near out-of-the-money} \\ 3 \text{ if } 0.975 < S/K \leq 1.025, & \text{at-the-money} \\ 4 \text{ if } 1.025 < S/K < 1.1, & \text{at-the-money near in-the-money} \\ 5 \text{ if } S/K \geq 1.1, & \text{in-the-money} \end{cases}$$

5.2.1 Testing the Implications of Black-Scholes Model Assumption

One of the basic assumptions of the Black-Scholes model is that the variance of return rate is known and constant. If the model is correct, and option and stock markets efficient and synchronous, the model assumption implies that the volatility implicit in option price also is stationary over time and across maturities and/or striking prices.

The regression function for the call-option price, expressed in terms of the Black-Scholes function , F(.), and a random error (ε_t), is

$$C_t = F(x_t) + \varepsilon_t ,$$

where

$$F(x_t) = S N(d_1) - K e^{-rT} N(d_2) ;$$

$$d_1 = \frac{1}{\sqrt{x_t T}}[\ln(S/K) + (r + .5x_t)T], \text{ and } d_2 = d_1 - \sqrt{x_t T} .$$

The index t stands for the time the option is priced; N(.), the cumulative standard normal distribution; S, the stock price adjusted for dividends; K, the striking price; r, the risk-free interest rate; T, the time-to-maturity; and x_t, the implicit volatility.

To test the implications of the Black-Scholes' assumption, the behavior of x_t with respect to both moneyness degree (k) and maturity month (τ) must be specified. Given that the model is correct and that option and stock markets are efficient and syschronous, the Black-Scholes model assumption implies that the volatility implicit in option price also is stationary over time and across maturities and/or striking prices. Thus, if these implications of model assumption are violated, there are three possible means of determining how x_t is related to k and to τ.

First, consider the situation in which the implicit volatility deviates from its true mean as a function of degree-of-moneyness, k. That is, the hypothesized model is defined as

$$C_t = F\left(\sigma^2 + \sum_{k=1}^{5} b_k g_{tk}\right) + \varepsilon_t , \qquad (5.1)$$

where σ^2 is the true mean of the implicit volatilities across the degree-of-moneyness, k; b_k for k = 1, 2,...,5 represents the deviation of the implicit volatility from its mean such that $\sum_{k=1}^{5} b_k = 0$; and

$$g_{tk} = \begin{cases} 1 \text{ if the moneyness degree is k, and} \\ 0 \text{ otherwise} \end{cases}$$

Furthermore, the model assumes that the independent ε_t has zero mean with variance: $E\{\varepsilon_t^2\} = \sum_{k=1}^{5} \omega_k g_{tk}$.

Similarly, the second instance occurs when x_t is unequal across maturity month, τ. Here, the pricing model is specified as

$$C_t = F\left(\sigma^2 + \sum_{\tau=1}^{5} c_\tau h_{\tau t}\right) + \varepsilon_t , \qquad (5.2)$$

where c_τ for τ = 1, 2,...,5 represents the deviation of implicit volatility from its mean across maturity month, τ, such that $\sum_{\tau=1}^{5} c_\tau = 0$; and

$$h_{t\tau} = \begin{cases} 1 \text{ if the maturity month is } \tau\text{, and} \\ 0 \text{ otherwise,} \end{cases}$$

with $E\{\varepsilon_t^2\} = \sum_{\tau=1}^{5} \Omega_\tau h_{t\tau}$.

Finally, the third model assumes that x_t is a function of both striking price and maturity month. That is,

$$C_t = F\left(\sigma^2 + \sum_{k=1}^{5}\sum_{\tau=1}^{5} d_{k\tau} g_{tk} h_{t\tau}\right) + \varepsilon_t \qquad (5.3)$$

where $d_{k\tau}$ measures the deviation of the implicit volatility from its mean across both moneyness degree and maturity month, with the restriction that $\sum_{k}^{5}\sum_{\tau}^{5} d_{k\tau} = 0$. The model also assumes that $E\{\varepsilon_t^2\} = \sum_{k=1}^{5}\sum_{\tau=1}^{5} \Gamma_{k\tau} g_{tk} h_{t\tau}$.

These three alternative models will be tested against the Black-Scholes model, whose assumption is that the implicit volatility is constant ($x_t = \sigma^2$) across k and τ. Thus, to adjust for the unequal option variances across option positions, the generalized nonlinear least squares is needed to fit the alternative models as well as the Black-Scholes. A sizable reduction in the sum of squares of residuals (SSE) indicates that the Black-Scholes assumption is inappropriate. Specifically, the F-test,

$$F = \frac{(SSE^{BS} - SSE^{i}) / (df^{BS} - df^{i})}{MSE^{i}} ,$$

is compared with the tabulated F-distribution at α level, with the upper degrees of freedom equal to the difference between the degrees of freedom from the SSE of the Black-Scholes and the i^{th} alternative model, and the lower degrees of freedom equal to those from the SSE of the i^{th} alternative model.

Because all models specified in the study have the same form as the Black-Scholes function, rejecting the Black-Scholes assumption in favor of the alternative models does not imply that the Black-Scholes model itself is useless in predicting option price level or that the stock volatility is described best by the Ito process. Data indicate only that model option prices will relate relatively closely to observed market option prices if a suitable set of implied volatilities is substituted into the Black-Scholes function.

5.2.2 Constructing a Robust Model

Although it has been well known since 1976 that implied volatilities vary across maturities and/or striking prices, most empirical studies compute a single volatility estimate on the basis of all options written on a stock at a certain time and use one such estimate to analyze option mispricing. Whaley (1982) and Stephan and Whaley (1990)[3], on the other hand, argue that maturity-specific implied stock volatilities are more appropriate than single implied volatility estimate sharingfor all options. Our study, however, assumes that the implied volatilities not only contain information on anticipated change in stock returns but also relate systematically to option position with respect to time-to-maturity and moneyness degree. The study therefore follows the results of statistical tests from the previous subsection, which strongly support the third alternative model.

To examine the difference between the Black-Scholes model prices and the observed market prices, previous empirical studies (see for example, MacBeth and Merville (1979), and Whaley (1982)) relate the difference between the market price and the Black-Scholes model price of the same option to the extent to which the option is in or out of the money, as measured by

$$m = \frac{S - Ke^{-rt}}{Ke^{-rt}}$$

[3]Their idea is based upon the argument of Patell and Wolfson (1979) that the standard deviation implicit in option price for a longer-lived option written on a stock is greater than that for a shorter-lived option if there is an anticipated information event, such as an earnings announcement, between the expirations of the two options. Thus, their estimated implied volatilities derive from a model similar to Model 5.2 of this study.

and/or by option maturities (T). If regression results indicate a significantly positive (negative) relation between the residual of the Black-Scholes model and the regressor(s), options would seem overpriced (underpriced) with respect to striking prices or maturities.

The significance of the effect of striking prices and maturities indicates, however, that

1) the observed option prices themselves are related closely to the degree-of-moneyness (m),
2) the observed option prices themselves are related closely to their maturities (T), and
3) the implied volatilities themselves are related closely to striking prices and to maturities.

Furthermore, MacBeth and Merville (1979) and Whaley (1982) report that the regression residuals also are correlated serially. These regression problems need correction if a meaningful conclusion about the systematic mispricing of the Black-Scholes model is to be arrive at. Both striking-price (m) and maturity (T) factors must be included as regressors in the regression model and allowed to change across moneyness degree (k) and maturity month (τ). Similarly, specification of implied volatilities must be such that they can vary across k and τ. Finally, an autoregressive process reflecting the serial correlation among option prices is included. Surprisingly, as will be seen in Section 5.3.2., this random component changes systematically across k and τ. Thus, the AR1 (ρ) process is specified in such a way that it can change additively across k and τ.

The regression function corresponding to the foregoing model description is

$$C_t \quad = \quad F\left(\sigma^2 + \sum_{i=1}^{4}\theta_i Z_{it} + \sum_{j=1}^{4}\phi_j W_{jt}\right) + \left(\alpha + \sum_{i=1}^{4}\lambda_i Z_{it} + \sum_{j=1}^{4}\gamma_j W_{jt}\right).(m_t - \overline{m})$$

$$+ \left(\beta + \sum_{i=1}^{4} \psi_i Z_{it} + \sum_{j=1}^{4} \varphi_j W_{jt}\right).(T_t - \overline{T}) \;+\; \varepsilon_t \;, \tag{5.4}$$

with

$$\varepsilon_t \;=\; \left(\rho + \sum_{i=1}^{4} \vartheta_i Z_{it} + \sum_{j=1}^{4} \upsilon_j W_{jt}\right).\varepsilon_{t-1} + \eta_t \;.$$

$\overline{m}$ and $\overline{T}$ are the means of m and T, respectively; Z_i's (their definitions are the example for GE options) and W_j's are

$$Z_1 = \begin{cases} 1 & \text{for Mar} \\ 0 & \text{for Apr,May,Jun}\,, \\ -1 & \text{for Sept} \end{cases} \qquad Z_2 = \begin{cases} 1 & \text{for Apr} \\ 0 & \text{for Mar,May,Jun}\,, \\ -1 & \text{for Sept} \end{cases}$$

$$Z_3 = \begin{cases} 1 & \text{for May} \\ 0 & \text{for Mar,Apr,Jun} \\ -1 & \text{for Sept} \end{cases} \;, \qquad Z_4 = \begin{cases} 1 & \text{for Jun} \\ 0 & \text{for Mar,Apr,May}\,, \\ -1 & \text{for Sept} \end{cases}$$

$$W_1 = \begin{cases} 1 \text{ if } \frac{S}{K} \le 0.9 \\ -1 \text{ if } \frac{S}{K} \ge 1.1 \\ 0 \text{ Otherwise} \end{cases} \;, \qquad W_2 = \begin{cases} 1 \text{ if } 0.9 < \frac{S}{K} \le 0.975 \\ -1 \text{ if } \frac{S}{K} \ge 1.1 \\ 0 \text{ Otherwise} \end{cases} \;,$$

$$W_3 = \begin{cases} 1 \text{ if } .975 < \frac{S}{K} \le 1.025 \\ -1 \text{ if } \frac{S}{K} \ge 1.1 \\ 0 \text{ Otherwise} \end{cases} \;, \text{and} \quad W_4 = \begin{cases} 1 \text{ if } 1.025 < \frac{S}{K} < 1.1 \\ -1 \text{ if } \frac{S}{K} \ge 1.1 \\ 0 \text{ Otherwise} \end{cases} \;.$$

Thus, Z_i's and W_j's have the same role as do g_k's and h_τ's in previous subsections in identifying options according to moneyness degree and maturity month. Model 5.4 assumes, however, that η_t is distributed identically, with zero mean and constant variance.

The stock return variance rate not a function of Z_i's and W_j's but implicit in option price is measured by σ^2. But, in practice, as mentioned in previous subsections, implicit variance (implied volatility) is related systematically to option position. Thus, Model 5.4 has introduced θ_i and ϕ_j to represent the deviation of implicit volatility from its constant value, σ^2, with respect to maturity month and moneyness degree, respectively. From the definition of Z_i's, θ_i for i = 1, 2, 3, 4, and $-\Sigma\theta_i$ represents the deviation of the implied volatility from σ^2 of the GE options maturing in March, April, May, June, and September, respectively. Likewise, ϕ_j for j = 1, 2, 3, 4, and $-\Sigma\phi_j$ measures the deviation of the implied volatility from σ^2 across moneyness degree, k, for k = 1, 2,..., 5, respectively.

The interpretation of the individual parameters α, λ_i, γ_j, β, ψ_i, φ_j, ρ, ϑ_i, and υ_j is similar to that for σ^2, θ_i, and ϕ_j. For example, deviation of the striking price effect (from its mean, α) across maturity month, τ for τ = 1, 2,..., 5, is represented by λ_i for i = 1, 2, 3, 4, and $-\Sigma\lambda_i$, respectively. On the other hand, deviation of the striking effect (from its mean, α) with respect to the degree-of-moneyness, k for k = 1, 2,..., 5, is measured by γ_j from j = 1, 2, 3, 4, and $-\Sigma\gamma_j$, respectively.

The most valuable information that can be extracted from the model is the partial effects of striking price and time-to-maturity, which are represented by the linear combination of α, λ_i, γ_j and β, ψ_i, φ_j for i, j = 1, 2, 3, 4, respectively. These two effects combined determine the mispricing of the model price, F(.). For example, according to the definition of Z's and W's and the meaning of the individual parameter α, λ_i, γ_j, β, ψ_i, and φ_j, the partial effect of time-to-maturity (across k and τ) on the options written on GE in March 1990 is determined according to the significance of elements in Table 5.1. Because the model is designed to examine the systematic pattern of striking price and maturity biases, it is constructed additively.

Table 5.1 The Maturity Effects Across Degree-of-Moneyness and Maturity Month

Mature in	March	April	May	June	September
S/K≤0.9	$\beta+\psi_1+\varphi_1$	$\beta+\psi_2+\varphi_1$	$\beta+\psi_3+\varphi_1$	$\beta+\psi_4+\varphi_1$	$\beta-\Sigma\,\psi+\varphi_1$
.9<S/K≤.975	$\beta+\psi_1+\varphi_2$	$\beta+\psi_2+\varphi_2$	$\beta+\psi_3+\varphi_2$	$\beta+\psi_4+\varphi_2$	$\beta-\Sigma\,\psi+\varphi_2$
.975<S/K≤1.025	$\beta+\psi_1+\varphi_3$	$\beta+\psi_2+\varphi_3$	$\beta+\psi_3+\varphi_3$	$\beta+\psi_4+\varphi_3$	$\beta-\Sigma\,\psi+\varphi_3$
1.025<S/K<1.1	$\beta+\psi_1+\varphi_4$	$\beta+\psi_2+\varphi_4$	$\beta+\psi_3+\varphi_4$	$\beta+\psi_4+\varphi_4$	$\beta-\Sigma\,\psi+\varphi_4$
S/K≥1.1	$\beta+\psi_1-\Sigma\,\varphi$	$\beta+\psi_2-\Sigma\,\varphi$	$\beta+\psi_3-\Sigma\,\varphi$	$\beta+\psi_4-\Sigma\,\varphi$	$\beta-\Sigma\,\psi-\Sigma\,\varphi$

From Table 5.1, the partial effects of time-to-maturity of out-of-the-money March, April, May, June, and September options depend upon the value of $\beta + \psi_i + \varphi_1$ for i = 1, 2, 3, 4, and $\beta - \Sigma\psi_i + \varphi_1$. Thus, the difference between the partial effects of the out-of-the-money March and May options can be determined by $\psi_1 - \psi_2$, which is, for example, also equal to the difference between the partial effects of the in-the-money March and May options. Thus, Model 5.4 is specified such that the effect of time-to-maturity (striking price, implied volatility, and serial correlation coefficient) of options with different maturities (moneyness degrees) is the same across degree-of-moneyness (maturity month).

By substituting (β, ψ, and φ) for (σ^2, θ, and ϕ), (α, λ, and γ), and (ρ, ϑ, and υ), respectively, Table 5.1 also applies for examinations of the systematic effects of implied volatility, striking price, and AR1 process.

For simplicity of notation, let

$$\sigma^2_{k\tau t} = \sigma^2 + \sum_{i=1}^{4}\theta_i Z_{it} + \sum_{j=1}^{4}\phi_j W_{jt} \quad ,$$

$$\alpha_{k\tau t} = \alpha + \sum_{i=1}^{4}\lambda_i Z_{it} + \sum_{j=1}^{4}\gamma_j W_{jt} \quad ,$$

$$\beta_{k\tau t} = \beta + \sum_{i=1}^{4}\psi_i Z_{it} + \sum_{j=1}^{4}\varphi_j W_{jt} \quad \text{, and}$$

$$\rho_{k\tau t} = \rho + \sum_{i=1}^{4} \vartheta_i Z_{it} + \sum_{j=1}^{4} \upsilon_j W_{jt} \quad .$$

then, Equation 5.4 can be rewritten as

$$C_t = F(\sigma^2_{k\tau t}) + \alpha_{k\tau t}(m_t - \overline{m}) + \beta_{k\tau t}(T_t - \overline{T}) + \rho_{k\tau t}\varepsilon_{t-1} + \eta_t \; . \qquad (5.5)$$

Not only can this model correct various kinds of regression problems faced by the conventional approach, many hypothesis testings of interest can be performed. Furthermore, the model permits estimation of implied volatilities in the cells that have, even, zero observations. The SYSNLIN procedure of SAS/ETS[4] can be applied easily to estimate parameters in Model 5.5.

To confirm the adequacy of the specified model and to confirm the invalidity of the Black-Scholes model assumption[5], four alternative models will be tested against Model 5.5:

$$C_t = F(\sigma^2_{k\tau t}) + \alpha_{k\tau t}(m_t - \overline{m}) + \beta_{k\tau t}(T_t - \overline{T}) + \rho\varepsilon_{t-1} + \eta_t \; , \qquad (5.6)$$

$$C_t = F(\sigma^2) + \alpha_{k\tau t}(m_t - \overline{m}) + \beta_{k\tau t}(T_t - \overline{T}) + \rho_{k\tau t}\varepsilon_{t-1} + \eta_t \; , \qquad (5.7)$$

$$C_t = F(\sigma^2_{k\tau t}) + \alpha_{k\tau t}(m_t - \overline{m}) + \beta_{k\tau t}(T_t - \overline{T}) + \eta_t \; , \text{ and} \qquad (5.8)$$

$$C_t = F(\sigma^2) + \alpha_{k\tau t}(m_t - \overline{m}) + \beta_{k\tau t}(T_t - \overline{T}) + \eta_t \quad . \qquad (5.9)$$

Models 5.7 and 5.9 support the Black-Scholes assumption; but the former (and also Models 5.5 and 5.6) allows option prices to be correlated serially. To determine the contribution of the AR1 process to the observed option prices, Model 5.8 is tested against Model 5.5. Surprisingly, in an examination of Models 5.5 and 5.6, not only is the superiority of Model 5.5 revealed but also the AR1 process changes systematically across maturities and striking prices.

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[4]SAS/ETS User's Guide, Version 5 Edition. Cary, NC: SAS Institute Inc., 1984.
[5]In the previous subsection, option prices are assumed to be independent across t but to share a common variance with respect to k and/or to τ. This subsection, however, relaxes the independent assumption.

Another advantage to applying Model 5.5 is that the model is so general that it can be used to examine other kinds of behavior of the systematic mispricing of Black-Scholes. For example, one may wish to know whether the deviation of observed and model prices across striking prices (striking price bias) is the same for May and June options, or whether the pattern of deviation across maturities (maturity bias) is the same for near-in and near-out of the money options. Model 5.5 also can be used to investigate the behavior of at-the-money implied volatilities. Most empirical studies claim that these estimates seem stable over option maturity. With the help of Model 5.5, this kind of hypothesized statement can be tested easily. In addition, Model 5.5 offers a means of estimating the implied volatility of stock return, σ^2, that is *independent* of trading time, striking price, and maturity. Thus, this stable estimated implied volatility may be a better indicator of stock volatility than is either the at-the-money implied volatility or the historical variance.

5.2.3 Data

The options data used for this study are taken from the Market Data Report of the CBOE. They consist of reported trades and quotes on the floor of the CBOE in March 1990 for options on AVON, GE, GM, IBM, COKE, and AT&T. Only call-option prices having a total contract volume of at least 3 and having been traded after 1,000 seconds after 9:00 are selected. The dividend information for these underlying assets comes from Standard and Poors Stock Record. The risk-free rate for each option maturity is computed using the average of bid and ask discounts of the Treasury Bill whose maturity most closely matches maturity of the option. The present value of dividends is computed based upon the yield on the Treasury Bill maturing near each ex-dividend date.

5.3 Results

5.3.1 Testing Implication of the Black-Scholes' Assumption

This section first presents the result of tests of the implication of the assumption underlying the Black-Scholes model, i.e., the stability of the implied volatility implicit in the option price across maturities, and/or striking prices. To test this assumption, the first step of the procedure is to use the nonlinear least squares procedure to calculate the estimated implied volatilities across maturity month (τ), and/or degree-of-moneyness (k). The second step is to reapply the nonlinear least squares technique, treating the residual mean squares from the first step as weights. These weights are used again to compute the implied volatility from the Black-Scholes model. This second step is required to ensure that the option variances from Models 5.1 to 5.3 are close to those from Black-Scholes. If the result from step one indicates, however, that option variances are equal, then it is unnecessary to follow step two. For example, the Mar 90 options on GE will be analyzed step by step, but for other options, the first step of the procedure will be omitted.

Step 1 According to Section 5.2.1, there are three models describing the behavior of implicit volatilities. The first states that the implicit volatility deviates from the true variance rate of return, σ^2, by the amount b_k across degree-of-moneyness, k, for k = 1, 2,...,5. That is,

$$C_t = F(x_t) + \varepsilon_t , \quad \text{with} \qquad E\{\varepsilon_t^2\} = \sum_{k=1}^{5} \varpi_k g_{tk} ,$$

where

$$x_t = \sigma^2 + \sum_{k=1}^{5} b_k g_{tk} .$$

Estimation results of this model are summarized in Table 5.2. According to Table 5.2, the estimated implied volatilities increase as the options go in the money. Because the estimated option variances, MSE, seem to increase as striking price decreases, except for the at-the-money near-in-the-money options, the table supports the models underlying assumption that option variances are unequal across striking prices, Thus, to test the hypotheses that

$$H_o: \; b_1 = b_2 = ... = b_5 = 0 \quad \text{and}$$
$$H_a: \; \text{at least one } b_k \neq 0 \;,$$

the weighted nonlinear least squares need to apply to both the Black-Scholes model and Model 5.1 if the option variances from the two models are to be equal.

Table 5.2 Nonlinear Least Squares Fitted to Model 1 For GE Options

S/K	≤ 0.9	0.9 < ≤ .975	.975 < ≤ 1.025	1.025 < < 1.1	≥ 1.1
$\hat{\sigma}_k^2$	.037828	.038976	.038994	.04901	.062631
SSE	1 .35783235	21.9628683	39.1567416	20.720607	4.416449
MSE	.0104448	.0175702	.0319125	.0215167	.0322368
d.f	130	1250	1227	963	137

Step 2 The weighted nonlinear least squares with weights equal to residual mean squares are fitted to all the alternative models, to test the homogeneity of volatility implicit in option price across option position. If each of hypothesis is satisfied, then the implication of the Black-Scholes model assumption cannot be rejected:

$$1) \quad H_o: \; b_1 = b_2 = ... = b_5 = 0 \;,$$
$$2) \quad H_o: \; c_1 = c_2 = ... = c_5 = 0 \quad \text{and}$$
$$3) \quad H_0: \; d_{k\tau} = 0 \quad \text{for all } k \text{ and } \tau \;.$$

The summary statistics with which to test the implication of the Black-Scholes assumption of constant implicit volatility across maturities and/or striking prices for Mar 90 options on GE, GM, IBM, ATT, AVON, and COKE are given in Tables 5.3 through 5.8, respectively. The F-computed's in Tables 5.3 through 5.8 are the calculated F test formula presented in Subsection 5.2.1. These statistics show that the implicit volatilities do vary across option maturities and/or exercise prices, because all the F-computed's are greater than the tabulated F statistics. As a result, all three alternative models are superior to the Black-Scholes. Thus, the implication of the Black-Scholes assumption is rejected strongly at the 5% significance level. All alternative models, however, have the same functional form as does the Black-Scholes. Thus, the usefulness of the Black-Scholes model price is undeniable.

Table 5.3 Nonlinear Weighted Least Squares for Testing the Black-Scholes Model Assumption: Mar 90 GE Options

	Black-Scholes (BS) vs the ith Alternative Model					
	BS	1	BS	2	BS	3
Σ SSE	4702.725	3707	4308.0397	3707	6265.197	3690
Σ d.f	3711	3707	3711	3707	3711	3690
F-computed	248.93		150.26		122.628	
F-table at $\alpha = .05$	2.37		2.37		1.57	

Table 5.4 Nonlinear Weighted Least Squares for Testing the Black-Scholes Model Assumption: Mar 90 GM Options

	Black-Scholes (BS) vs the ith Alternative Model					
	BS	1	BS	2	BS	3
Σ SSE	5452.597	3417	3729.581	3417	8942.449	3400
Σ d.f	3421	3417	3421	3417	3421	3400
F-computed	508.89		78.15		263.926	
F-table at $\alpha = .05$	2.37		2.37		1.57	

Table 5.5 Nonlinear Weighted Least Squares for Testing the Black-Scholes Model Assumption: Mar 90 IBM Options

	Black-Scholes (BS) vs the ith Alternative Model					
	BS	1	BS	2	BS	3
Σ SSE	26426.260	25414	29009.019	25414	35774.634	25397
Σ d.f	25418	25414	25418	25414	25418	25397
F-computed	1012.26		3595.019		10377.634	
F-table at $\alpha = .05$	2.37		2.37		1.57	

Table 5.6 Nonlinear Weighted Least Squares for Testing the Black-Scholes Model Assumption: Mar 90 ATT Options

	Black-Scholes (BS) vs the ith Alternative Model					
	BS	1	BS	2	BS	3
Σ SSE	3301.214	2792	3135.583	2792	6625.986	2775
Σ d.f	2796	2792	2796	2792	2796	2775
F-computed	127.30		85.89		183.38	
F-table at $\alpha = .05$	2.37		2.37		1.57	

Table 5.7 Nonlinear Weighted Least Squares for Testing the Black-Scholes Model Assumption: Mar 90 AVON Options

	Black-Scholes (BS) vs the ith Alternative Model					
	BS	1	BS	2	BS	3
Σ SSE	3133.032	2431	3158.854	2431	5886.815	2413
Σ d.f	2435	2431	2435	2431	2435	2413
F-computed	175.51		181.96		157.90	
F-table at $\alpha = .05$	2.37		2.37		1.57	

Table 5.8 Nonlinear Weighted Least Squares for Testing the Black-Scholes Model Assumption: Mar 90 COKE Options

	Black-Scholes (BS) vs the ith Alternative Model					
	BS	1	BS	2	BS	3
Σ SSE	1663.148	1541	1599.783	1541	2077.275	1525
Σ d.f	1545	1541	1545	1541	1545	1525
F-computed	30.537		14.69		27.61	
F-table at $\alpha = .05$	2.37		2.37		1.57	

These statistical test results support the visual investigation of Latane and Rendleman (1976), Schmalensee and Trippi (1978), MacBeth and Merville (1979), Beckers (1980), and Rubinstein (1985). These authors find that their results are inconsistent with the Black-Scholes framework, but the model is, nevertheless, valuable in predicting market option prices. It may be that the model is somewhat insensitive to violations of the nonstationary assumption.

5.3.2 Investigating the Mispricing of the Black-Scholes Model

Because the test results and the results of other empirical studies indicate that implicit volatilities not only contain information regarding anticipated change in stock returns but also relate systematically to the option position with respect to time-to-maturity and/or striking price, further analysis of the thesis will be based upon the structure of Model 5.3.

Studies of the systematic mispricing of the Black-Scholes model typically have regressed the option prediction errors on option time-to-maturity (T), and/or degree-of-moneyness (m). This tradition explains the systematic deviation patterns by examining the sign and the significance of regression coefficients. But the technique, in turn, makes test statistics unreliable and interpretations of coefficients meaningless. This problem occurs because the strong correlation of prediction errors (regression residuals) with maturities and/or striking prices implies that market option prices themselves are highly correlated with these regressors. Johnston (1984) thus states that the omission of these relevant regressors from the original model (that producing the prediction errors) yields biased coefficient estimates. Furthermore, the conventional inference procedures are undermined because the disturbance variance cannot be estimated correctly. One way to correct this problem is to include these important regressors (maturities and striking prices) in the model. Models 5.8 and 5.9 represent these corrections, but the latter is based on the Black-Scholes assumption of constant volatility:

$$C_t = F(\sigma^2_{k\tau t}) + \alpha_{k\tau t}(m_t - \overline{m}) + \beta_{k\tau t}(T_t - \overline{T}) + \eta_t \text{ , and} \qquad (5.8)$$

$$C_t = F(\sigma^2) + \alpha_{k\tau t}(m_t - \overline{m}) + \beta_{k\tau t}(T_t - \overline{T}) + \eta_t \text{ .} \qquad (5.9)$$

Another problem common in this field of study is that market option prices are correlated serially. The consequences of ignoring this problem and of using least squares procedures to estimate option parameters (see, for example, Judge et al., 1982) are that the least squares estimator will, in general, be inefficient, and the regression variance estimate biased. Thus, to improve efficiency of estimate, and at the same time to reduce regression variance bias, Models 5.8 and 5.9 are modified:

$$C_t = F(\sigma^2_{k\tau t}) + \alpha_{k\tau t}(m_t - \overline{m}) + \beta_{k\tau t}(T_t - \overline{T}) + \rho_{k\tau t}\varepsilon_{t-1} + \eta_t \text{ ,} \qquad (5.5)$$

$$C_t = F(\sigma^2_{k\tau t}) + \alpha_{k\tau t}(m_t - \overline{m}) + \beta_{k\tau t}(T_t - \overline{T}) + \rho\varepsilon_{t-1} + \eta_t \text{ , and} \qquad (5.6)$$

$$C_t = F(\sigma^2) + \alpha_{k\tau t}(m_t - \overline{m}) + \beta_{k\tau t}(T_t - \overline{T}) + \rho_{k\tau t}\varepsilon_{t-1} + \eta_t \text{ .} \qquad (5.7)$$

Again, the Black-Scholes assumption is double checked by testing Model 5.5 against Model 5.7 (and also Model 5.8 against Model 5.9). This time, however, the independence assumption of market option prices is relaxed. The most interesting part of testing the systematic mispricing of the Black-Scholes is the test between Models 5.5 and 5.6. If the former is statistically favorable, then another kind of systematic correlation of option price with option position arises. That is, the unobserved variation, that is left over after extracting the expiration effect and the exercise opportunity from market option price, is related systematically to option positions. Rubinstein (1985) considered this phenomenon evidence that a composite model should be built. Furthermore, the bias observed in any period should be correlated with the level of certain macroeconomic variables such as stock-market price levels, volatilities, and interest rate levels.

Because Model 5.5 contains a great number of parameters that need to be estimated, convergence of the model depends upon the number of observations in each option position, as well as upon the total number of observations. After Model 5.5 is applied to GE, GM, IBM, ATT, AVON, and COKE options, only options based on the first three underlying securities yield convergent results. The total number of observations of GE, GM, and IBM options are 3,712, 3,422, 25,419, respectively (after the contract volume less than 3 and trading time of the first 1,000 seconds after 9:00 a.m. ETS are screened out); options on ATT, AVON, and COKE have only 2,797, 2,436, and 1,546 observations, respectively. Thus, analysis in this section focuses on GE, GM, and IBM options.

The general statistics of Models 5.5 through 5.9 for GE, GM, and IBM options are summarized in Tables 5.9 through 5.11. According to these tables, the SSE of Model 5.9 is almost twice that of Model 5.5, whereas the residual degrees of freedom of Models 5.9 and 5.5 are nearly the same.

Table 5.9 General Statistics for GE Options

Model	d.f.(error)	SSE	MSE	R^2	D.W.
5	3676	28.8726	.007854	.9981	2.211
6	3684	30.3526	.008239	.9980	2.281
7	3684	29.4915	.008005	.9980	2.205
8	3685	49.2439	.01336	.9967	0.773
9	3693	52.2593	.01415	.9965	0.739

D.W. is the Durbin-Watson test statistics.

Table 5.10 General Statistics for GM Options

Model	d.f.(error)	SSE	MSE	R^2	D.W.
5	3386	25.1367	.007424	.9975	2.254
6	3394	25.2715	.007446	.9975	2.253
7	3394	26.5039	.007809	.9974	2.246
8	3395	39.0085	.01149	.9961	0.828
9	3403	41.3299	.01215	.9959	0.825

D.W. is the Durbin-Watson test statistic.

Table 5.11 General Statistics for IBM Options

Model	d.f.(error)	SSE	MSE	R^2	D.W.
5	25383	319.3292	.01258	.9981	2.321
6	25391	350.7858	.01382	.9979	2.428
7	25391	321.6600	.01267	.9981	2.319
8	25392	600.0923	.02363	.9964	0.725
9	25400	610.4125	.02403	.9964	0.718

D.W. is the Durbin-Watson test statistics.

Thus, the variation of the random component of Model 5.9 seems much greater than that of Model 5.5, as indicated by MSE size. Furthermore, Durbin-Watson test statistics shown in Tables 5.9 through 5.11 demenstrate that the option prices explained by Models 5.8 and 5.9 are (positively) serially correlated. This problem vanishes, however, when the AR1 process is introduced to Models 5.5, 5.6, and 5.7.

Given the information in Tables 5.9 through 5.11, the performance of each model compared with that of Model 5.5, which is more general, can be examined. Results are presented in Table 5.12. The superiority of Model 5.5 to other models, especially to Models 5.8 and 5.9, is clear because the magnitude of all computed F values is much greater than that of the tabulated ones. These differences are enormous when the performance of Model 5.5 is compared with that of either Model 5.8 or Model 5.9.

The performance tests between Models 5.5 and 5.7 (the second row of Table 5.12) and also between Models 5.8 and 5.9 (these F-computed's are 28.21, 25.25, and 54.59, respectively, for GE, GM, and IBM) confirm rejection of the Black-Scholes assumption. The most striking result occurs when Model 5.5 is tested against Model 5.6 (the first row of Table 5.12). The AR1 process also varies across option maturities and/or exercise prices.

Table 5.12 Examining the Performance of the Alternative Models to Model 5.5

Model 5 Vs	Computed F-Statistics for Options On			Tabulated F Test
	GE	GM	IBM	
Model 6	23.55	2.27	312.55	1.94
Model 7	9.85	23.02	23.16	1.94
Model 8	288.18	212.56	2479.72	1.88
Model 9	175.15	128.31	1361.05	1.67

At this point, sufficient evidence supports the specifications of Model 5.5, which not only best explains the observed market option prices, but also improves the efficiency of estimates and reduces the bias of regression variance. The conventional approach, on the other hand, ignores the serial correlation of option prices and their correlation with option positions. Thus, the coefficient estimates obtained from the conventional approach are biased and inefficient, as is the regression variance.

The next analysis is based on the estimated coefficients of regression Equation 5.4 (or Model 5.5), especially those of striking price and maturity effect. The partial effect of striking prices adjusted for mean $(m - \bar{m})$ or maturities adjusted for mean $(T - \bar{T})$ across in-the-money degree, k, and maturity month, τ, on market option prices, however, is expressed as the linear combination of these coefficients, as shown in Table 5.1, Section 5.2.1. Thus, examinations of the systematic deviation of the partial effect of striking price and maturity in Tables 5.13 through 5.15 use the format shown in Table 5.1. Additionally, to examine these effects as well as the estimated implied volatilities and the coefficients of moving average across maturity month and degree-of-moneyness, it is convenient to gather all information in the same table. Thus, Tables 5.13 through 5.15 are designed such that each cell in the table indicates the position of options, whereas the area inside the cell is divided into four parts, as shown in Figure 5.1. The upper left corner of Figure 5.1 contains the estimated implied volatility; the upper right and lower left corners represent the partial effect of striking price $(\hat{\alpha}_{k\tau})$ and maturity $(\hat{\beta}_{k\tau})$, respectively.

$$\boxed{\begin{array}{cc} \hat{\sigma}^2_{k\tau} & \hat{\alpha}_{k\tau} \\ \hat{\beta}_{k\tau} & \hat{\rho}_{k\tau} \end{array}}$$

Figure 5.1 Position of Estimated Parameters

The estimated serial correlation coefficient $(\hat{\rho}_{k\tau})$ is in the lower right corner of the cell. These estimates are perfectly identical to the restricted nonlinear least squares estimation procedure, according to which $\hat{\sigma}^2_{k\tau}$'s are treated as known constants. Thus, when the effect of maturity (striking price) is treated as constant, the sign of $\hat{\alpha}_{k\tau}$ $(\hat{\beta}_{k\tau})$ partly reflects the direction of mispricing of the Black-Scholes model. The results of these estimates for options on GE in March 1990 are presented in Table 5.13.

As can be seen in Table 5.13, the estimated implied volatilities of at-the-money options are quite close to at-the-money near out-of-the-money options but these estimates are much smaller than those obtained from the at-the-money near in-the-money options. At-the-money near in-the-money options provide the greatest estimated implied volatilities, whereas out-of-the-money options yield the smallest. The magnitude of the estimated implied volatilities decreases as time-to-maturity increases, except in September.

Keeping time-to-maturity constant, Table 5.13 shows that the effect of striking price increases with maturity month. All else equal, however, maturity effect declines with striking price. The effect of striking price is greatest for at-the-money options. Out-of-the-money options yield the smallest striking-price effect but greatest time-to-maturity effect.

The serial correlation coefficients increase across maturity month except options expiring in May. In-the-money options yield the smallest estimates of these coefficients, whereas at-the-money options provide the greatest.

The estimated implied volatilities, the striking price and maturity effect, and the estimated serial correlation coefficients for GM options are reported in Table 5.14.

According to Table 5.14, the estimated implied volatilities are greatest for the in-the-money options, whereas the in-the-money near out-of-the-money options yield the smallest estimates. The estimated implied volatilities for the at-the-money and the at-the-money near in-the-money options are approximately the same.

The effect of striking price seems to increase with expiration. The at-the-money options provide the smallest estimates of the exercise price effect. The effect of time-to-maturity is negative for the at-the-money near in-the-money and for the in-the-money options. In May, the maturity effect is also negative and lowest. This effect becomes greater for the short- and the long-maturity options.

Table 5.13 The Effect of Striking Price and Maturity: GE Options

Mature in	March		April		May		June		September	
S/K ≤ 0.9	.0318*	-2.17*	.0223*	-1.99*	.0239*	-.87	.0198*	-.87	.0234*	1.85*
	(.0049)	(.63)	(.0042)	(.64)	(.0042)	(.66)	(.0038)	(.59)	(.0039)	(.64)
	1.53	.563*	.86	.599*	1.26	.235*	3.97*	.611*	2.11*	.689*
	(.45)	(.102)	(.55)	(.098)	(1.11)	(.111)	(.45)	(.097)	(.47)	(.093)
0.9 < S/K ≤ 0.975	.0451*	-.42	.0356*	-.25	.0372*	.87*	.0331*	1.38*	.0367*	3.60*
	(.0025)	(.32)	(.0010)	(.27)	(.0009)	(.31)	(.0009)	(.19)	(.0033)	(.26)
	.09	.704*	-.58	.739*	-.18	.375*	2.53*	.752*	.68	.829*
	(.15)	(.043)	(.31)	(.034)	(.99)	(.063)	(.36)	(.031)	(.41)	(.029)
0.975 < S/K ≤ 1.025	.0449*	1.25	.0355*	1.42*	.0370*	2.55*	.0329*	3.05*	.0365*	5.27*
	(.0026)	(.68)	(.0012)	(.68)	(.0007)	(.72)	(.0013)	(.67)	(.0034)	(.74)
	.20	.792*	-.47	.828*	-.07	.464*	2.64*	.840*	.79	.918*
	(.19)	(.038)	(.30)	(.027)	(.97)	(.058)	(.42)	(.031)	(.44)	(.031)
1.025 < S/K < 1.1	.0532*	-1.32*	.0437*	-1.15*	.0453*	-.02	.0412*	.48	.0448*	2.70*
	(.0023)	(.35)	(0012)	(.34)	(.0013)	(.38)	(.0016)	(.32)	(.0034)	(.42)
	-.79*	.422*	-1.47*	.459*	-1.06	.094	1.64*	.471*	-.21	.548*
	(.11)	(.028)	(.29)	(.030)	(.99)	(.063)	(.39)	(.037)	(.37)	(.040)
S/K ≥ 1.1	.0509*	-.43*	.0414*	-.25	.0429*	.87	.0388*	1.37	.0425*	3.59*
	(.0040)	(.17)	(.0031)	(.24)	(.0030)	(.32)	(.0031)	(.19)	(.0046)	(.37)
	-1.11	.404*	-1.78*	.441*	-1.38	.076	1.33*	.453*	-.52	.530*
	(.21)	(.059)	(.33)	(.062)	(1.01)	(.082)	(.41)	(.061)	(.37)	(.062)

* The estimated parameter is significantly different from zero at 95% confidence level.
The number in ( ) is the systemptotic standard deviation.

Table 5.14 The Effect of Striking Price and Maturity: GM Options

Mature in	March		April		May		June		September	
S/K ≤ 0.9	.0586*	-.35	.0648*	.29	.0969*	3.25*	.0890*	2.76*	.0959*	5.59*
	(.0115)	(.67)	(.0116)	(.76)	(.0116)	(.91)	(.0124)	(.85)	(.0094)	(.80)
	-.44	.629*	-1.65*	.598*	-3.83	.541*	-3.27*	.592*	-2.03*	.428*
	(.62)	(.107)	(.67)	(.108)	(2.08)	(.151)	(.81)	(.106)	(.44)	(.093)
0.9 < S/K ≤ 0.975	.0204*	-3.49*	.0266*	-2.85*	.0587*	.11	.0508*	-.39	.0577*	2.45*
	(.0046)	(.46)	(.0027)	(.42)	(.0031)	(.58)	(.0033)	(.41)	(.0091)	(.58)
	1.03*	.795*	-.17	.764*	-2.35	.707*	-1.79*	.758*	-.55	.594*
	(.25)	(.049)	(.36)	(.050)	(2.03)	(.111)	(.49)	(.046)	(.63)	(.058)
0.975 < S/K ≤ 1.025	.0421*	-7.81*	.0482*	-7.16*	.0803*	-4.21*	.0725*	-4.70*	.0793*	-1.87*
	(.0036)	(.80)	(.0016)	(.78)	(.0023)	(.89)	(.0032)	(.76)	(.0092)	(.78)
	-.04	.726*	-1.25*	.695*	-3.43	.639*	-2.86*	.689*	-1.63*	.526*
	(.18)	(.030)	(.34)	(.038)	(2.02)	(.114)	(.47)	(.036)	(.55)	(.055)
1.025 < S/K < 1.1	.0473*	-1.55*	.0535*	-.89	.0856*	2.05*	.0778*	1.56*	.0846*	4.39*
	(.0041)	(.58)	(0023)	(.49)	(.0023)	(.68)	(.0031)	(.51)	(.0093)	(.70)
	-1.09*	.595*	-2.29*	.564*	-4.48*	.507*	-3.91*	.558*	-2.68*	.394*
	(.12)	(.025)	(.29)	(.032)	(2.02)	(.114)	(.48)	(.034)	(.56)	(.059)
S/K ≥ 1.1	.1357*	-2.54*	.1418*	1.89*	.1739*	1.06	.1661*	.57	.1729*	3.39*
	(.0138)	(.45)	(.0137)	(.42)	(.0139)	(.63)	(.0147)	(.46)	(.0162)	(.66)
	-3.54*	.606*	-6.93*	.575*	-6.93*	.518*	-6.36*	.569*	-5.13*	.406*
	(.44)	(.041)	(2.05)	(.045)	(2.05)	(.118)	(.67)	(.045)	(.58)	(.065)

* The estimated parameter is significantly different from zero at 95% confidence level.
The number in ( ) is the systemptotic standard deviation.

Table 5.15 The Effect of Striking Price and Maturity: IBM Options

Mature in	March		April		May		July		October	
S/K ≤ 0.9	.0561*	-1.77*	.0358*	.54*	.0346*	2.69*	.0402*	-.02	.0534*	.12
	(.0023)	(.24)	(.0023)	(.19)	(.0024)	(.33)	(.0024)	(.26)	(.0042)	(.38)
	-.92*	.254*	.46	.656*	9.81*	.658*	-1.46*	.656*	-2.94*	.629*
	(.36)	(.130)	(.42)	(.129)	(.89)	(.130)	(.39)	(.129)	(.58)	(.129)
0.9 < S/K ≤ 0.975	.0484*	.05	.0282*	1.28*	.0269*	4.52*	.0325*	1.81*	.0457*	1.94*
	(.0006)	(.19)	(.0004)	(.18)	(.0008)	(.31)	(.0008)	(.17)	(.0036)	(.28)
	.37*	.555*	1.75*	.957*	11.11*	.959*	-.16	.957*	-1.64*	.931*
	(.12)	(.012)	(.27)	(.007)	(.82)	(.009)	(.19)	(.008)	(.52)	(.010)
0.975 < S/K ≤ 1.025	.0373*	-.86*	.0171*	.38*	.0159*	3.61*	.0214*	.89*	.0346*	1.03*
	(.0005)	(.18)	(.0006)	(.16)	(.0009)	(.32)	(.0009)	(.24)	(.0036)	(.38)
	-.42*	.591*	.96*	.993*	10.31*	.995*	-.96*	.993*	-2.44*	.966*
	(.08)	(.010)	(.25)	(.002)	(.83)	(.006)	(.20)	(.004)	(.52)	(.009)
1.025 < S/K < 1.1	.0524*	.92*	.0321*	2.15*	.0309*	5.39*	.0365*	2.67*	.0496*	2.81*
	(.0005)	(.12)	(0002)	(.08)	(.0007)	(.29)	(.0007)	(.19)	(.0036)	(.35)
	-.03	.231*	1.35*	.632*	10.71*	.635*	-.56*	.632*	-2.04*	.606*
	(.07)	(.009)	(.24)	(.009)	(.82)	(.011)	(.17)	(.009)	(.48)	(.012)
S/K ≥ 1.1	.0617*	-.42*	.0414*	.82*	.0402*	4.05*	.0458*	1.34*	.0589*	1.47*
	(.0016)	(.14)	(.0016)	(.09)	(.0017)	(.29)	(.0016)	(.17)	(.0039)	(.35)
	-.57*	-.005	.81*	.396*	10.17*	.399*	-1.10*	.396*	-2.58*	.369*
	(.19)	(.035)	(.28)	(.033)	(.84)	(.034)	(.19)	(.034)	(.49)	(.035)

* The estimated parameter is significantly different from zero at 95% confidence level.
The number in ( ) is the systemptotic standard deviation.

Serial correlation coefficients seem to decline as maturity increases, and the at-the-money near out-of-the-money options yield the greatest estimates.

The estimated implied volatilities, the striking price and maturity effect, and the estimated serial correlation coefficients for IBM options are reported in Table 5.15. According to this table, estimates of implied volatilities are smallest in May and become greater for the longer and the shorter maturity options. With respect to degree-of-moneyness, these estimates are smallest at at-the-money and increase as options become in- or out-of-the-money.

The striking price effect is greatest and positive for options maturing in May. The March options yield the smallest estimates of the striking-price effect. The effect of time-to-maturity is greatest in positive value for the May options and declines as maturity becomes either shorter or longer. The maturity effect is approximately the same for the at-the-money and the in-the-money options.

The estimated coefficients of serial correlation are greater for longer maturity options. These estimates are greatest for the at-the-money options and decrease as options become in- or out-of-the-money.

Given the information in Tables 5.13 through 5.15, the mean of the deviation between observed and model option prices, F(.), for options on GE, GM, and IBM can be computed across degree-of-moneyness and maturity month. Because the estimated coefficients of Model 5.5 must be identical to those from the restricted nonlinear least squares procedure when σ^2, θ_i, and ϕ_j are substituted by their optimal value, the mean of the deviation between observed and model price,

$$E\left\{C_t - F(\sigma^2_{k\tau t})\right\} ,$$

can be estimated by

$$\hat{\alpha}_{k\tau t}(m_t - \overline{m}) + \hat{\beta}_{k\tau t}(T_t - \overline{T}) \cdot$$

Thus, on average, the magnitude of the mispricing of the model price across option positions is computed by averaging these terms across k and τ. That is, for k and $\tau = 1, 2, \ldots, 5$ the mispricing of the model price is measured as

$$\hat{\alpha}_{k\tau t}(\overline{m}_{k\tau} - \overline{m}) + \hat{\beta}_{k\tau t}(\overline{T}_{k\tau} - \overline{T}) \quad , \tag{5.10}$$

where $\overline{m}_{k\tau}$ and $\overline{T}_{k\tau}$ are means of m and T across k and τ, respectively.

The mispricing of the model price, F(.), as measured by the Expression 5.10 for options on GE is reported in Table 5.16. According to this table, the average mispricing for all June and most September options is positive. Thus, on average, the model underprices GE options whose maturities are medium to long. Furthermore, Table 5.16 also indicates strongly that. on average, the at-the-money near in-the-money and the in-the-money options with short maturities also are overvalued compared with the model price (given that the Black-Scholes function, F(.), is corrected). Additionally, the model correctly prices the most at the-money short maturity options. Thus, the striking price bias for the GE options is such that the at-the-money near in-the-money and the in-the-money options are overpriced. On the other hand, the maturity bias is such that the medium to long maturity options also are overpriced.

According to previous empirical studies, for example MacBeth and Merville (1979), Whaley (1982), Gultekin, Rogaski, and Tinic (1982), and Levy and Byun (1987), striking price and maturity bias in options written on a certain stock share the patterns of those options written on other securities. Thus, it is expected that the striking price and the maturity biases of GM and IBM options would have basically the same direction as those of GE options. Nevertheless, results for GM and IBM options disagree with those of previous research.

Table 5.16 The Systematic Deviation of Market and Model Prices: GE Options

Mature in	March	April	May	June	September
$S/K \le 0.9$	.028 (.057)	N/A	N/A	.443* (.083)	.603* (.187)
$.9 < S/K \le .975$	.006 (.021)	.043* (.019)	-.052* (.021)	.172* (.029)	.126 (.144)
$.975 < S/K \le 1.025$	-.017 (.029)	.021 (.024)	-.014 (.013)	.192* (.030)	.432* (.146)
$1.025 < S/K < 1.1$	.067* (.008)	.034* (.016)	.016 (.029)	.182* (.039)	.186 (.115)
$S/K \ge 1.1$	.093* (.015)	.074* (.031)	N/A	.369* (.034)	.411* (.112)

*The estimated parameter is significantly different from zero at 95% confidence level.
The number in ( ) is the asymptotic standard deviation.
N/A, not applicable, indicates that there is no option sample to compute the mean of m and T in the cell.

The systematic mispricing of GM options is presented in Table 5.17. The means of the mispricing of June and September options all are significantly negative. The direction of maturity bias of the medium to long maturity GM options therefore is opposite that of the maturity bias of GE options. Furthermore, the at-the-money short maturity GM options are overpriced, whereas the same GE options are priced close to the model value. As maturity increases, mispricing mean becomes increasingly negative. The at-the-money (.9 < S/K < 1.1) short maturity options are, on average, overvalued.

For IBM options, as reported in Table 5.18, medium to long maturity options are, on average, underpriced, a result similar to that for GM options. But the direction of mispricing for the at-the-money short maturity IBM options is quite different from that of mispricing for the same GE or GM options. The at-the-money near out-of-the-money IBM options whose maturities are short are underpriced, whereas the at-the-money near in-the-money options are overpriced. Additionally, the at-the-money IBM options maturing in March are, on average, overpriced, whereas the at-the-money April options are undervalued.

Table 5.17 The Systematic Deviation of Market and Model Prices: GM Options

Mature in	March	April	May	June	September
S / K ≤ 0.9	.107 (.068)	N / A	N / A	-.712* (.174)	-1.309* (.195)
.9 < S / K ≤ .975	.218* (.040)	.229* (.029)	-.002 (.048)	-.135* (.058)	-.359 (.244)
.975 < S / K ≤ 1.025	.044* (.022)	.027* (.011)	-.011* (.002)	-.321* (.050)	-.649* (.203)
1.025 < S / K < 1.1	.119* (.009)	.122* (.019)	.018 (.021)	-.293* (.042)	-.724* (.186)
S / K ≥ 1.1	.124* (.017)	-.022 (.045)	N / A	-.556* (.113)	-1.428* (.206)

*The estimated parameter is significantly different from zero at 95% confidence level.
The number in ( ) is the asymptotic standard deviation.
N/A, not applicable, indicates that there is no option sample to compute the mean of m and T in the cell.

In the empirical tests of option mispricing, *within each study*, the bias with respect to exercise price or time-to-maturity always exhibited a consistent pattern, *regardless of the underlying security.* These biases typically fall into one of three categories. 1) In-the-money (out-of-the-money) model estimates tend to be higher (lower) than market prices. Black (1975), Gultekin et al. (1982), and Sterk (1983) report these striking price biases. 2) In-the-money (out-of-the-money) model estimates tend to be lower (higher) than market prices. MacBeth and Merville (1979, 1980) report these striking price biases. 3) Model estimates tend to be lower than market prices for near-to-maturity options, according to Black (1975) and Whaley (1982).

Model 5.5 and the foregoing criteria indicate that neither striking price nor maturity bias seems systematic across underlying stock. The same conclusion is applied for the estimated implied volatilities and for the serial correlation coefficients. But a certain underlying stock indicates that the mispricing of the Black-Scholes model is not negligible even though the model price has been adjusted for the instability of the estimated implied volatility and for the possibility of serial correlation among option prices, for all option classes. The bias patterns of option samples is summarized next.

- Maturity bias for GM and IBM options is such that the medium to long maturity options are undervalued. But this bias direction is reversed for GE options. Furthermore, the short maturity GE and GM options tend to be overpriced.
- Striking price bias for GE options is such that the at-the-money near in-the-money and the in-the-money options are overvalued.

Table 5.18 The Systematic Deviation of Market and Model Prices: IBM Options

Mature in	March	April	May	June	September
$S/K \le 0.9$	N/A	.061* (.023)	.075 (.041)	-.302* (.092)	-1.359* (.275)
$.9 < S/K \le .975$	-.033* (.0009)	-.077* (.008)	.241* (.031)	-.095* (.046)	-.830* (.256)
$.975 < S/K \le 1.025$	.033* (.007)	-.015* (.004)	.416* (.034)	-.209* (.047)	-1.161* (.249)
$1.025 < S/K < 1.1$	.047* (.003)	.095* (.005)	.707* (.043)	.065 (.038)	-.753* (.227)
$S/K \ge 1.1$	.002 (.013)	.108* (.012)	N/A	-.038* (.043)	N/A

*The estimated parameter is significantly different from zero at 95% confidence level.
The number in ( ) is the asymptotic standard deviation.
N/A, not applicable, indicates that there is no option sample to compute the mean of m and T in the cell.

Moreover, empirical studies usually use the at-the-money implied volatility as the estimate for the anticipated volatility of the stock rate of return because the estimates seem stable over option maturity, especially when time-to-maturity is at least 90 days to expiration.

Our study shows, however, that within the class of the at-the-money itself (the at-the-money near in- or out-of-the-money or the most at-the-money), the estimated implied volatilities do differ. Thus, no one estimated implied volatility seems suited to explaining the pricing behavior of the whole series of option prices. For example, the estimated implied volatilities of the most at-the-money and the at-the-money near in-the-money GM options (see Table 5.14) are twice the estimated implied volatilities of the at-the-money near out-of-the-money options. Furthermore, the mispricing behavior of the at-the-money short maturity options also are different. As can be seen from Table 5.18, the direction of the mispricing of the at-the-money near in- and out-of-the-money is perfectly opposite.

5.4 Robust Model Vs Conventional Approaches

Previous studies used two main approaches to examine the systematic mispricing of the Black-Scholes model option price. The first applies regression techniques by regressing the deviation between the market and the Black-Scholes model price on the extent to which option is in- or out-of-the-money (m) and/or the time-to-maturity (T) of the option contract. The other directly computes the mean of the difference between the market and the model price across moneyness degree and expiration month. Because Model 5.5 represents the relation between the mispricing and the extent to which option is in- or out-of-the-money and also the time-to-maturity across maturity month and degree-of-moneyness, the model is, in fact, a combination of the two approaches.

As mentioned in Section 5.2.2, regressing the option residual on variables affecting the option market price biases the coefficient estimates. The consequences of using least squares when disturbances are autocorrelated are that the least squares estimator will, generally, be inefficient and the estimate of the regression variance biased. Thus, a meaningful conclusion regarding the effect of striking price and maturity is drawn with difficulty from the regression approach (for details, see Johnston, 1984 and Judge et al., 1982).

The first part of this section will discuss the problem facing the simple regression technique, a problem arising from the model, namely the inability to distinguish option with respect to degree-of-moneyness (or maturity month). This problem may lead to misinterpretation of regression results. The second part of this section will discuss the disadvantages of the second approach.

Consider the results obtained by regressing the difference, R, between observed and model option prices of GM options on the extent to which option is in- or out-of-the-money, m, and also on m and time-to-maturity, T. In such an instance, the optimal values of the implied volatilities of Model 5.5 are used to proxy the implied volatilities.

$$\hat{R} = \underset{(.006)}{-.083} + \underset{(.090)}{.280}\,m\,, \qquad D.W. = .112$$

$$\hat{R} = \underset{(.005)}{.225} - \underset{(.054)}{.933}\,m - \underset{(.022)}{1.801}\,T\,, \qquad D.W. = .282$$

where the numbers in parenthesis are standard deviations of coefficient estimates.

The first regression results suggest that as options go in-the-money, mispricing increases. It is expected that, on average, the in-the-money options will be overpriced. These results contradict the findings in Table 5.17. From the table, the average mispricing with respect to degree-of-moneyness can be calculated by multiplying each number in the table by its corresponding number of observations and by averaging out across maturity months. These averages are presented in Table 5.19, in which the number beneath the average mispricing in each cell is the number of observations in that cell. From the table, the majority of market option prices, including in-the-money options, are underpriced. The plot of residuals supports this bias pattern, as shown in Figure 5.2.

The second regression results also mislead the direction of bias exhibited in the GM options. The model states that by keeping the effect of time-to-maturity constant, model prices tend to be greater than market prices as options go in-the-money. Because each column of Table 5.17 (or Table 5.19) represents the average mispricing across striking price by fixing maturity month (which also reflects time-to-maturity), it is expected from the regression results that average mispricing in each column decreases as stock-to-exercise-price ratio increases. Consider Figure 5.3 which is the plot of average mispricing against degree-of-moneyness (k) at different levels of maturity month. The figure indicates that only June and September options tend to be underpriced as striking price increases.

Table 5.19 The Average Mispricing Across Maturity Months: GM Options

Mature in	March	April	May	June	Sept	Average
$S/K \le 0.9$	.107 2	N / A	N / A	-.712* 49	-1.309* 58	-1.04
$.9 < S/K \le .975$	.218* 25	.229* 287	-.002 45	-.135* 352	-.359 150	-.035
$.975 < S/K \le 1.025$	.044* 450	.027* 171	-.011* 12	-.321* 230	-.649* 107	-.123
$1.025 < S/K < 1.1$	.119* 603	.122* 369	.018 34	-.293* 195	-.724* 8	.045
$S/K \ge 1.1$	.124* 148	-.022 43	N / A	-.556* 52	-1.428* 32	-.21

*The estimated parameter is significantly different from zero at 95% confidence level.

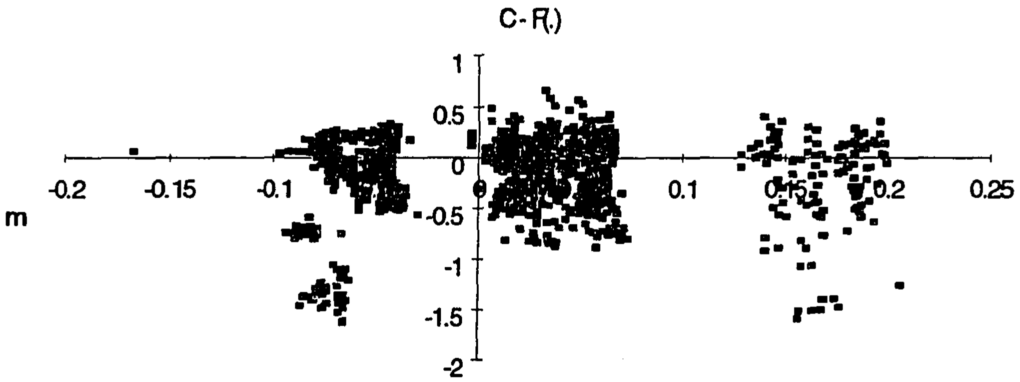

Figure 5.2 The Mispricing of GM Options

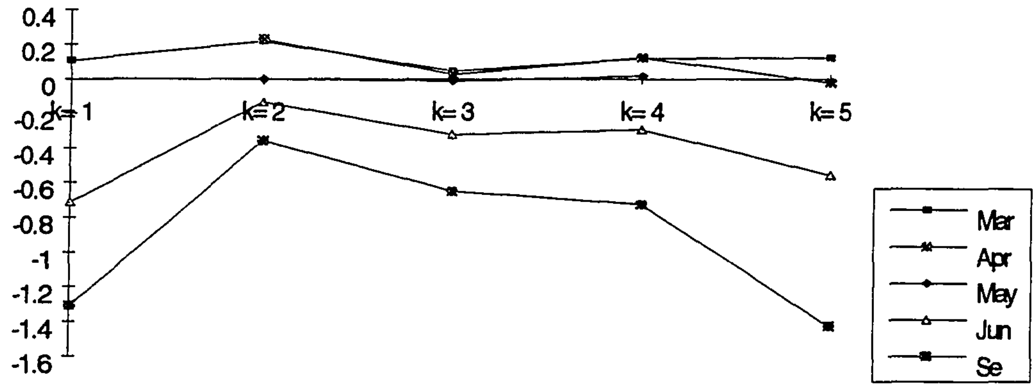

Figure 5.3 The Average Mispricing of GM Options Across Degree-of-Moneyness

Although these options represent only 36% of GM option samples, they influence the slope of the entire regression line. Thus, the inability of the simple regression technique to identify options across both striking price and maturity may make regression results unrealistic.

Another approach used by most studies to analyze option mispricing is that of directly classifying options according to maturity month and degree-of-moneyness and then computing average mispricing. That is, this technique calculates

$$\overline{R}_{k\tau} = \frac{\sum_{t=1}^{T_{k\tau}} (C_{k\tau t} - F_t(\hat{\sigma}^2_{k\tau}))}{T_{k\tau}}$$

for $k = 1,2,\ldots,5$; and $\tau = 1,2,\ldots,5$; $t = 1,2,\ldots,T_{k\tau}$. The technique has included the unexplained random component in the average mispricing. If, on average (across k, and τ), the random component has a zero mean and the effects of striking price, maturity, and model price, F(.), are orthogonal, then the directions of mispricing from this approach and from those of Model 5.5 may be the same. But the precision of estimates obtained from this approach is less than that obtained from Model 5.5. In other words, the standard deviation of the average mispricing from this approach is expected to be greater that of the robust model. As can be seen from Table 5.20, the direction of mispricing agrees with results in Table 5.17. But the standard deviations of the estimates (other than the out-of-the-money September, and the out-of-the-money near in-the-money March options, which represent only 13% of the entire sample) obtained from this approach are much greater than those in Table 5.17.

Table 5.20 The Systematic Deviation of Market and Model Prices of GM Options Obtained from the Average Mispricing Approach

Mature in	March	April	May	June	September
$S/K \le 0.9$	.043* (.013)	N / A	N / A	-.806* (.045)	-1.309* (.144)
$.9 < S/K \le .975$	.065* (.012)	.219* (.047)	-.057 (.089)	-.109 (.0805)	-.369 (.092)
$.975 < S/K \le 1.025$	.045 (.117)	.042 (.105)	.040 (.073)	-.337* (.120)	-.637* (.118)
$1.025 < S/K < 1.1$	.120 (.125)	.120 (.114)	.026 (.075)	-.294 (.155)	-.720* (.117)
$S/K \ge 1.1$	.108 (.127)	-.024 (.201)	N / A	-.493* (.200)	-1.449* (.095)

*The estimated parameter is significantly different from zero at 95% confidence level.
The number in ( ) is the asymptotic standard deviation.
N/A, not applicable, indicates that there is no option sample to compute the mean of m and T in the cell.

5.5 Graphical Performance of Robust vs Black-Scholes Model

The test statistics in Table 5.12 indicate clearly the superiority of Model 5.5 to other models designed to reduce residual sum of squares. Furthermore, the ability of the model to identify the mispricing behavior of the Black-Scholes function is quite impressive compared with that of the two conventional approaches. Yet these results do not confirm that the difference between the observation and the prediction of Model 5.5 exhibits no systematic pattern with respect to exercise price (m) or time-to-maturity (T). It, therefore, would be of interest to investigate whether Model 5.5 systematically over- or underprices options across option positions.

Actually, the construction of Model 5.5 and the estimation technique used in the study guarantee that the residuals of Model 5.5 are orthogonal to Z_i.m, w_j.m, Z_i.T, and w_j.T. Thus, theoretically, the residuals of Model 5.5 scatter around zero at any level of m and T. Furthermore, because the SSE of Model 5.5 is much smaller than that of the Black-Scholes model, the residuals from Model 5.5 should be closer to zero than is the deviation of the actual and the Black-Scholes model prices.

The computation of the Black-Scholes model price in this section follows the standard approach. That is, the at-the-money implied volatility is used as the proxy for the variance rate of stock return and substituted into the Black-Scholes formula. These estimates are .038994, .05269, and .033622 for GE, GM, and IBM options, respectively.

The residual plots[6] of Model 5.5 against both m and T in Figures 5.4 through 5.39 suggest the absence of systematic bias of Model 5.5 in predicting call option prices at any level of m and T as expected for all option samples. Furthermore, variations in the residuals are much smaller than those in prediction errors of the Black-Scholes model. On the other hand, the Black-Scholes model tends to underestimate the deep in- and the deep out-of-the-money options ($m < -0.1$ and $m > 0.1$, respectively). But when m assumes a value between -0.1 and -0.05, options seem undervalued, and this pattern seems to reverse when m has a value of between 0.05 and 0.1. This mispricing behavior mimics the findings of MacBeth and Merville (1979), except that their samples consist of only options whose degree of in-the-money (m) is approximately greater than -0.1. The maturity bias is weaker than the striking-price bias.

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[6] Because the computer software allows at most 4,000 samples to be plotted in the same graph, each plot for the IBM options contains 4,000 observations, except in Figures 5.24, 5.25, 5.38, and 5.39. These four plots contain the remainder of the data, or 1,419 options.

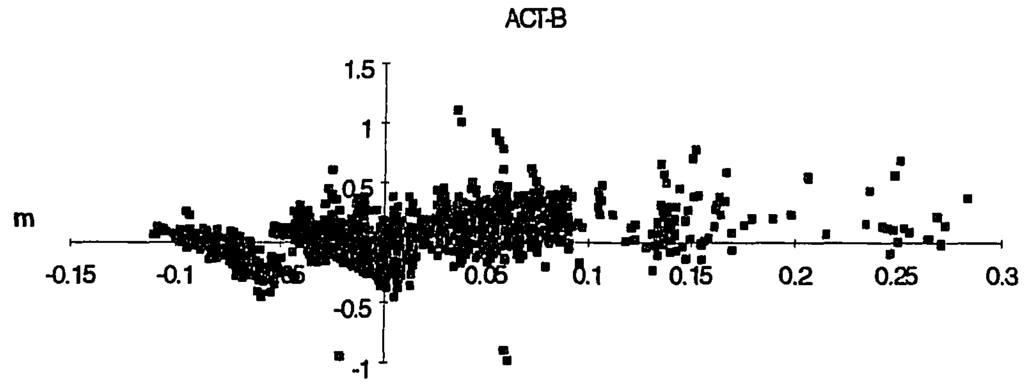


Figure 5.4 Deviation of the Observed from the BS Model Prices against m (GE)

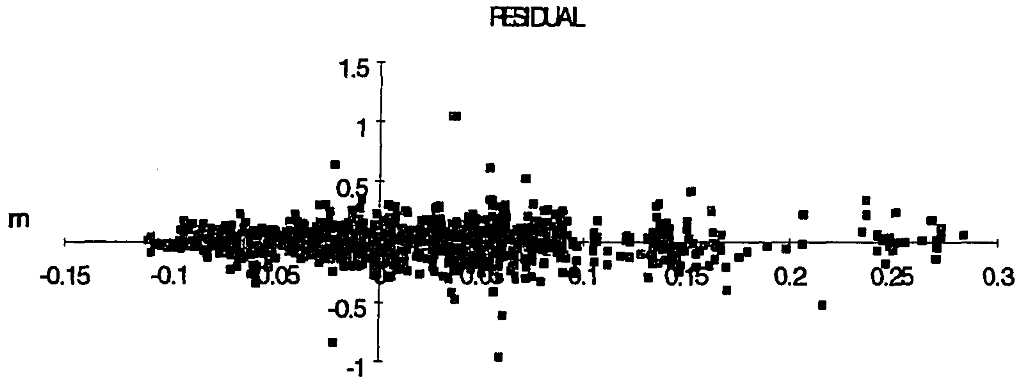


Figure 5.5 Residuals of Model 5 against m (GE)

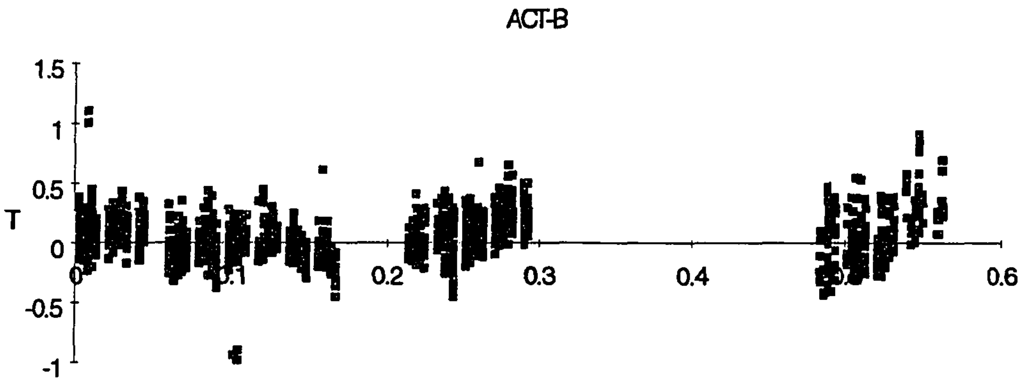


Figure 5.6 Deviation of the Observed from the BS Model Prices against T (GE)

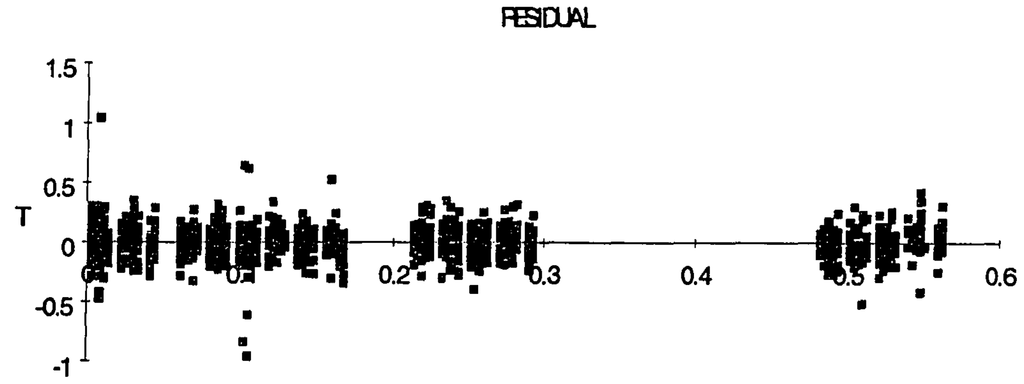


Figure 5.7 Residuals of Model 5 against T (GE)

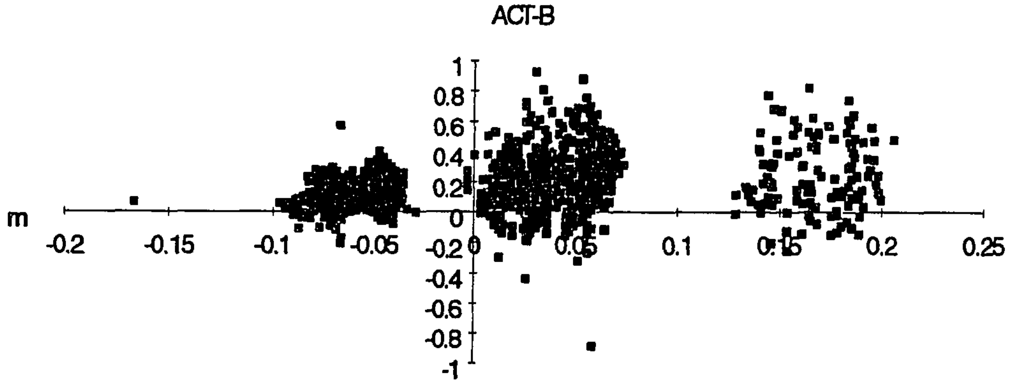


Figure 5.8 Deviation of the Observed from the BS Model Prices against m (GM)

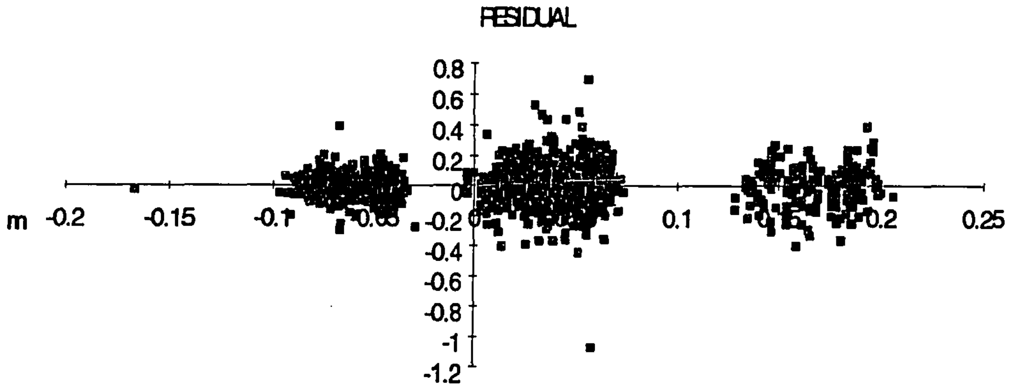


Figure 5.9 Residuals of Model 5 against m (GM)

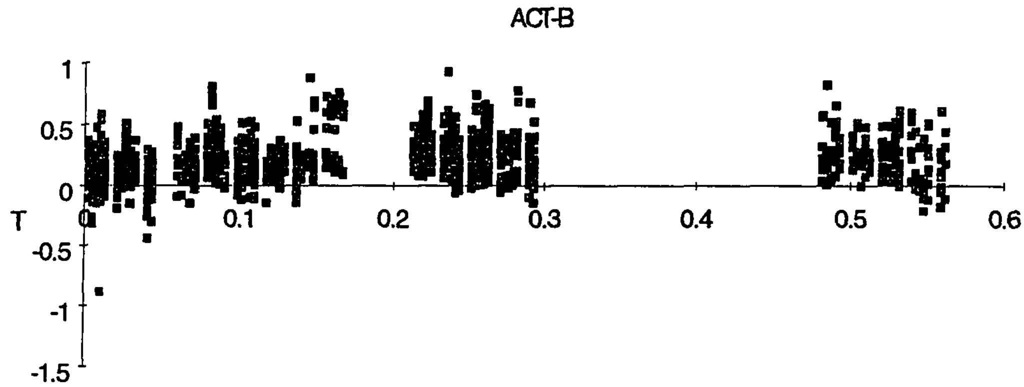


Figure 5.10 Deviation of the Observed from the BS Model Prices against T (GM)

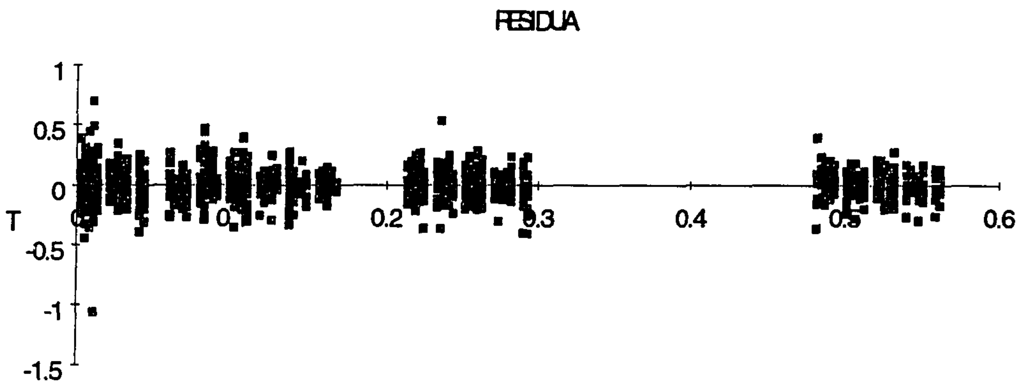


Figure 5.11 Residuals of Model 5 against T (GM)

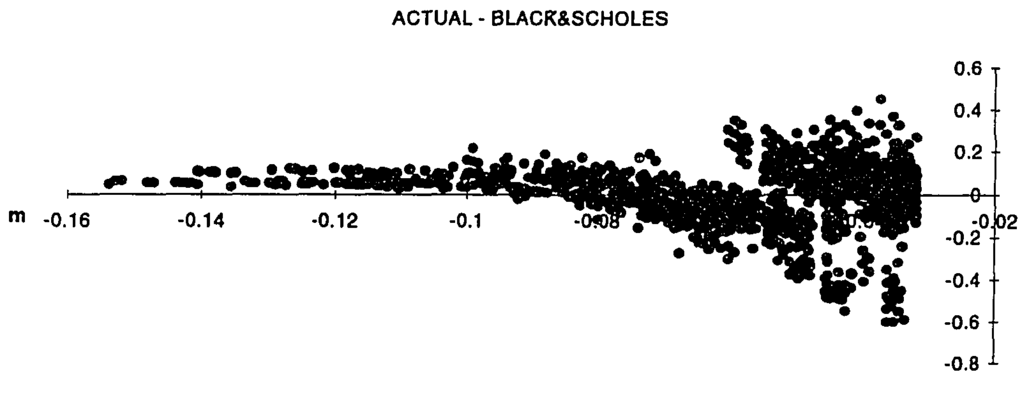


Figure 5.12 Deviation of the Observed from the BS Model Prices against m (IBM)

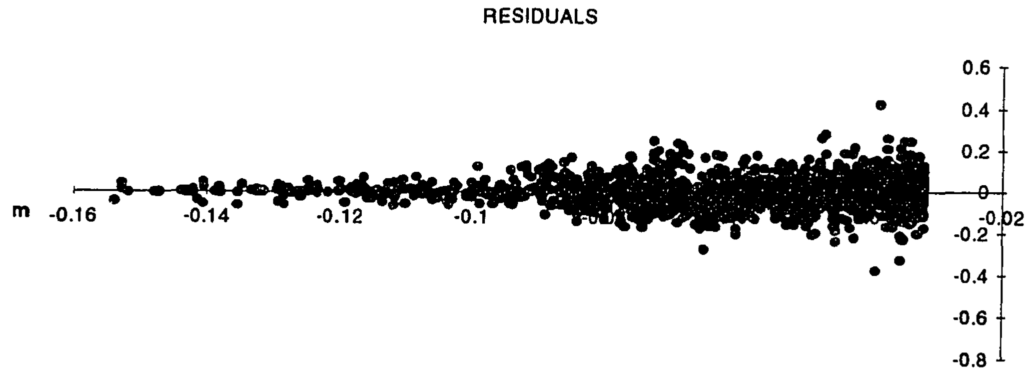


Figure 5.13 Residuals of Model 5 against m (IBM)

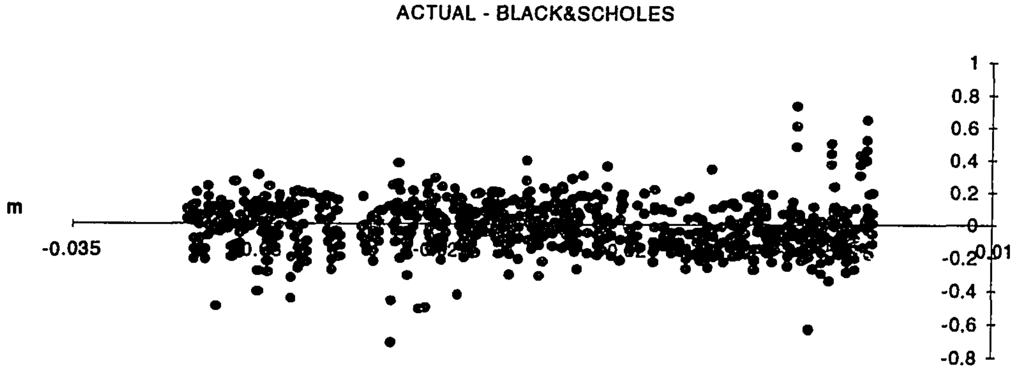


Figure 5.14 Deviation of the Observed from the BS Model Prices against m (IBM)

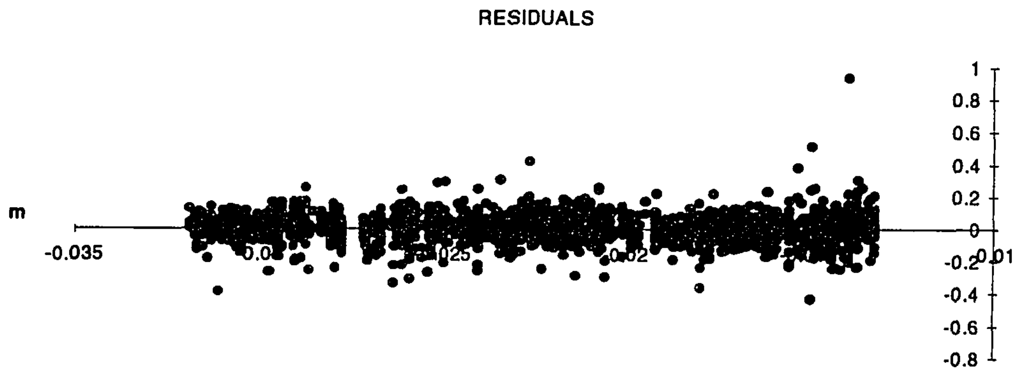


Figure 5.15 Residual of Model 5 against m (IBM)

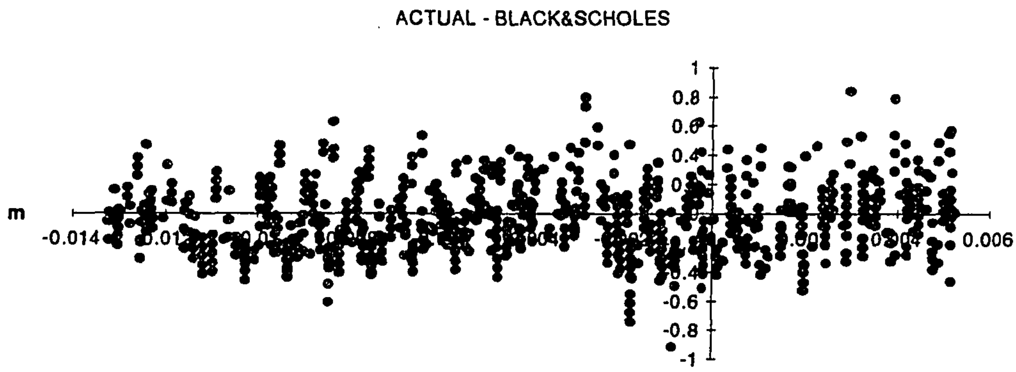


Figure 5.16 Deviation of the Observed from the BS Model Prices against m (IBM)

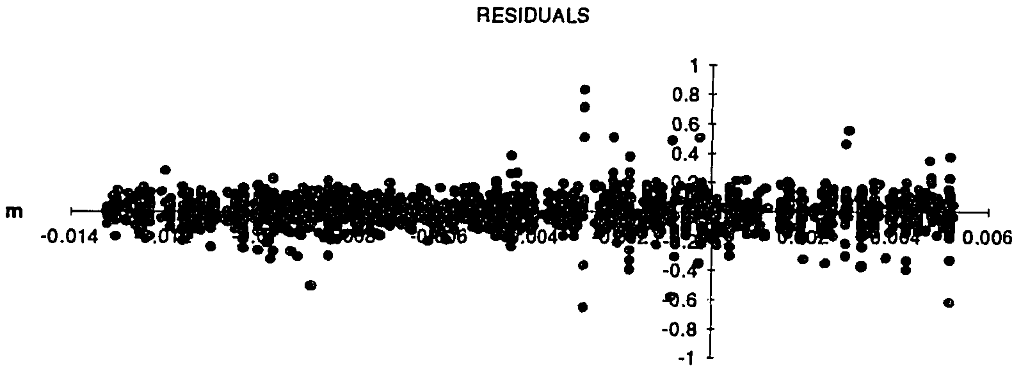


Figure 5.17 Residuals of Model 5 against m (IBM)

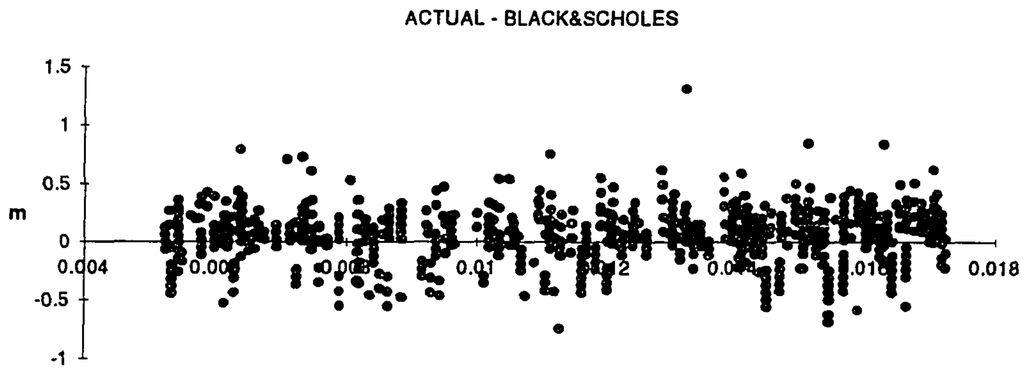


Figure 5.18 Deviation of the Observed from the BS Model Prices against m (IBM)

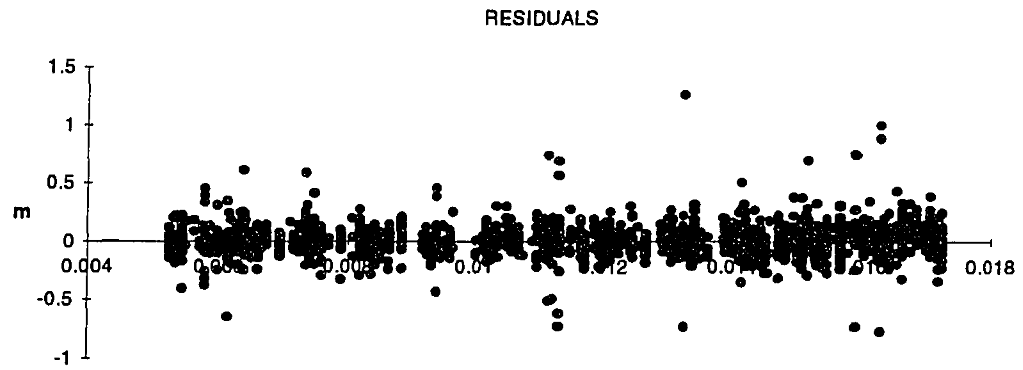


Figure 5.19 Residuals of Model 5 against m (IBM)

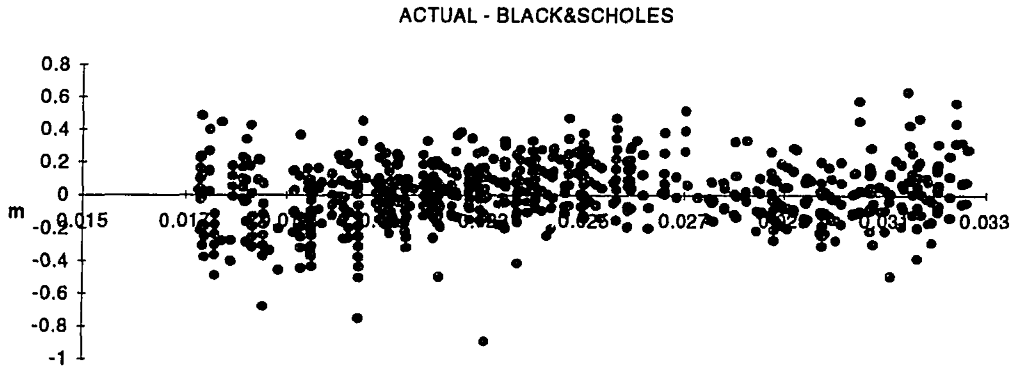


Figure 5.20 Deviation of the Observed from the BS Model Prices against m (IBM)

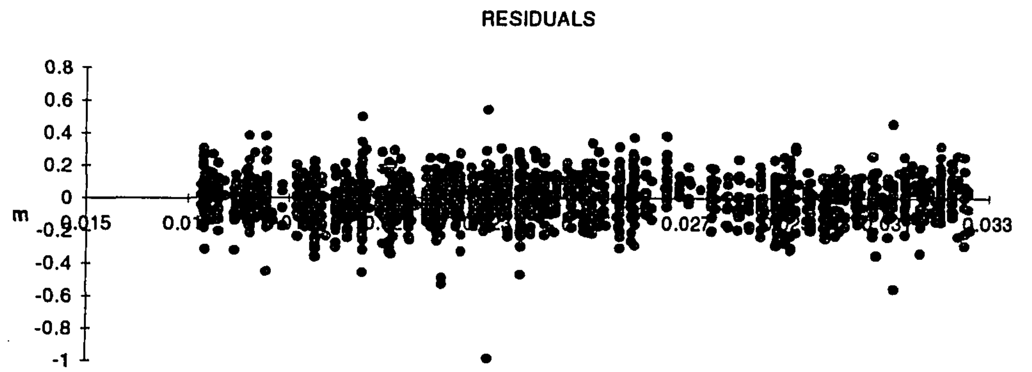


Figure 5.21 Residuals of Model 5 against m (IBM)

ACTUAL - BLACK&SCHOLES

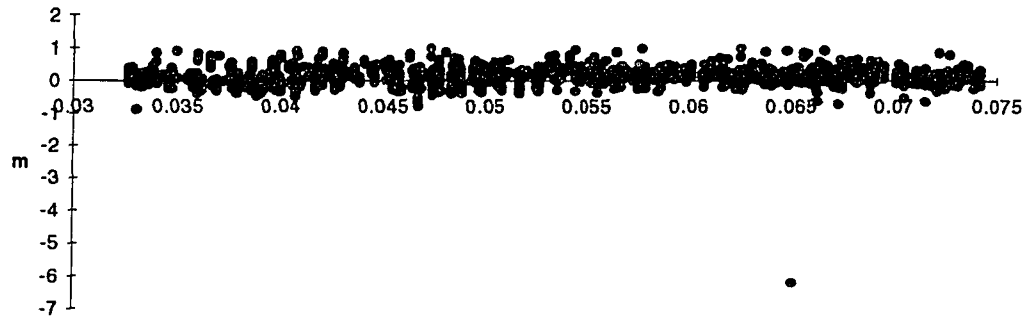


Figure 5.22 Deviation of the Observed from the BS Model Prices against m (IBM)

RESIDUALS

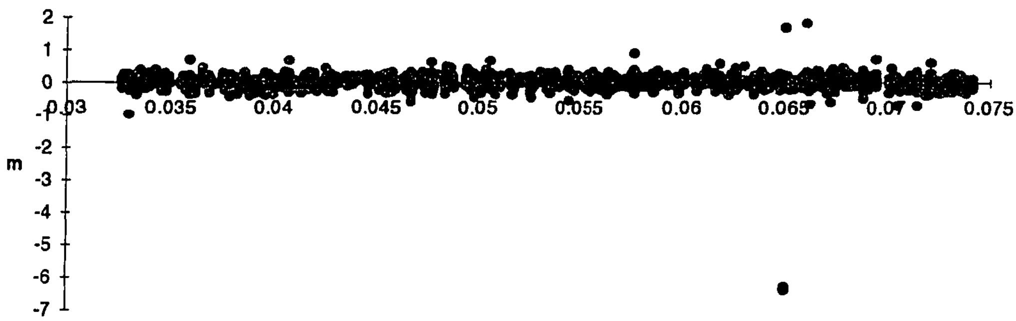


Figure 5.23 Residuals of Model 5 against m (IBM)

ACTUAL - BLACK&SCHOLES

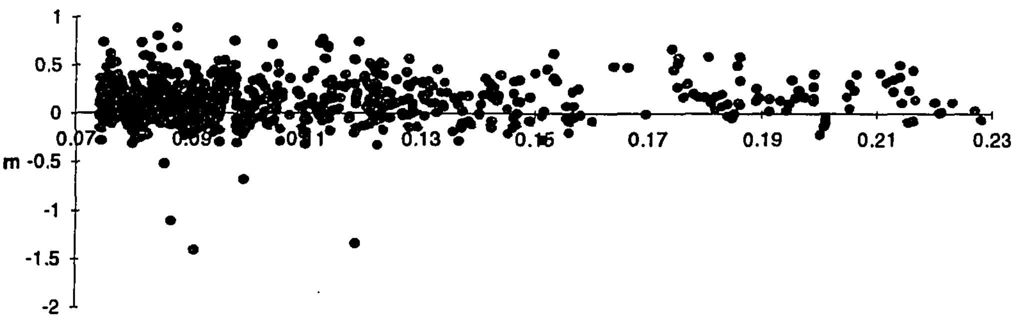


Figure 5.24 Deviation of the Observed from the BS Model Prices against m (IBM)

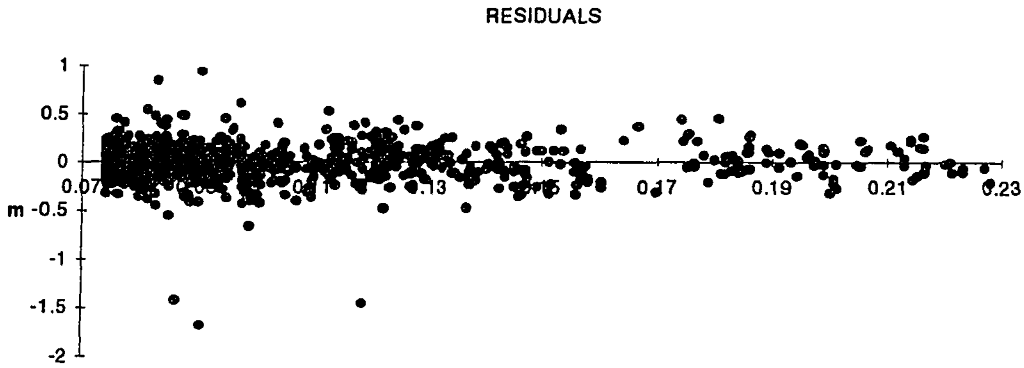


Figure 5.25 Residual of Model 5 against m (IBM)

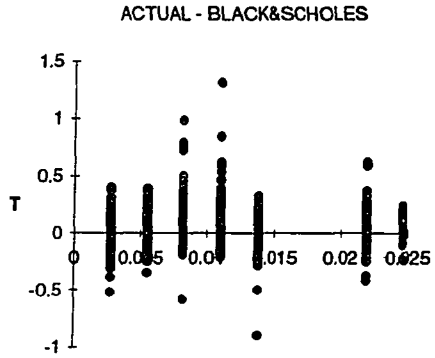


Figure 5.26 Deviation of the Observed from BS Model Prices against T (IBM)

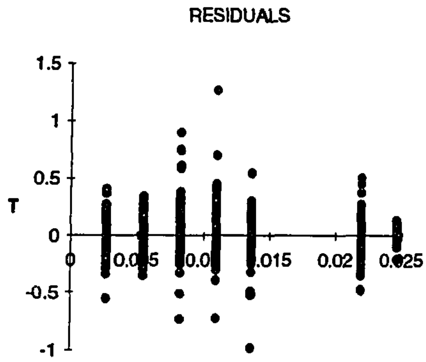


Figure 5.27 Residuals of Model 5 against T (IBM)

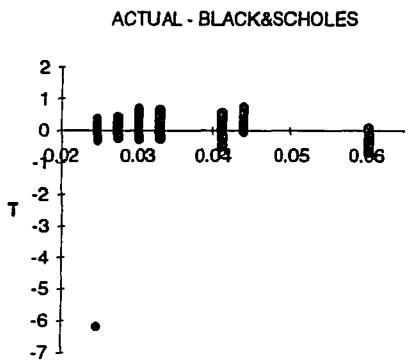


Figure 5.28 Deviation of the Observed from BS Model Prices against T (IBM)

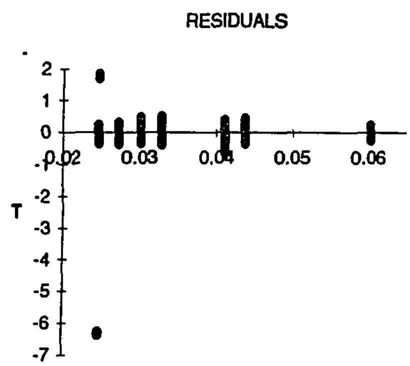


Figure 5.29 Residuals of Model 5 against T (IBM)

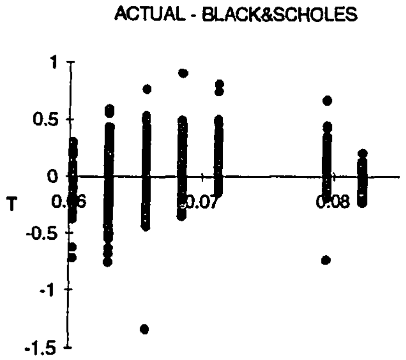


Figure 5.30 Deviation of the Observed from BS Model Prices against T (IBM)

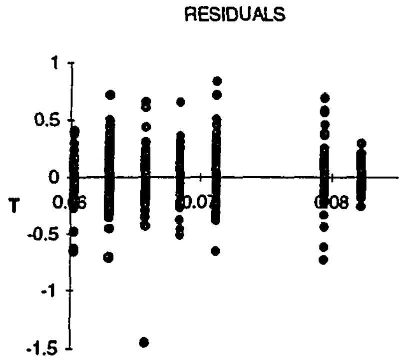


Figure 5.31 Residuals of Model 5 against T (IBM)

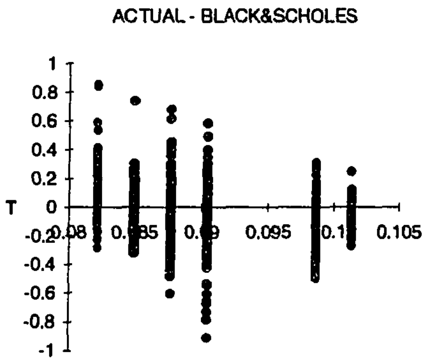


Figure 5.32 Deviation of the Observed from BS Model Prices against T (IBM)

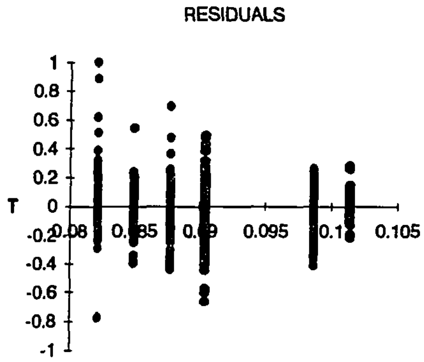


Figure 5.33 Residuals of Model 5 against T (IBM)

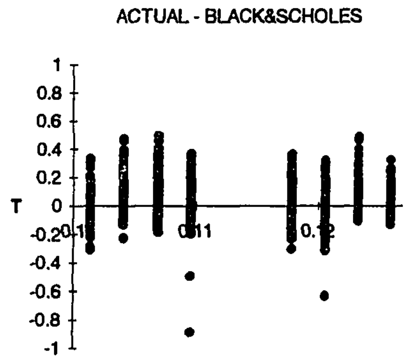


Figure 5.34 Deviation of the Observed from BS Model Prices against T (IBM)

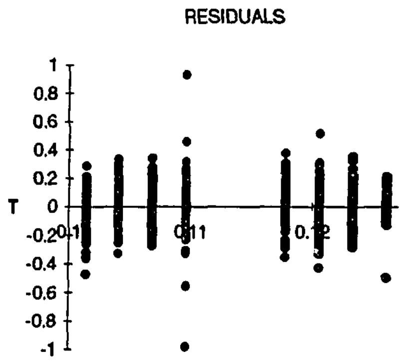
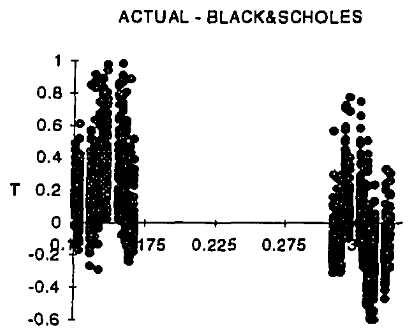


Figure 5.35 Residuals of Model 5 against T (IBM)



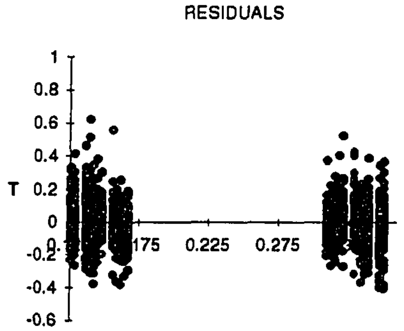
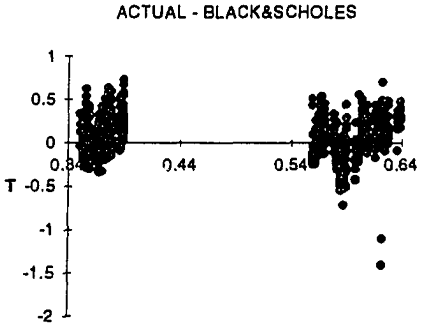


Figure 5.36 Deviation of the Observed from BS Model Prices against T (IBM)

Figure 5.37 Residuals of Model 5 against T (IBM)



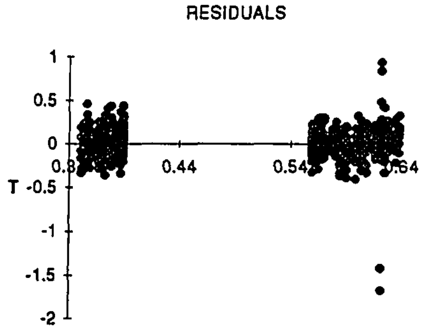


Figure 5.38 Deviation of the Observed from BS Model Prices against T (IBM)

Figure 5.39 Residuals of Model 5 against T (IBM)

To confirm the results of residual plots, the prediction of Model 5.5 and the actual call price within the range of option samples are plotted against the number of observations in Figures 5.40, 5.42, 5.44, 5.46, 5.48, 5.50, and 5.52. The plots of the Black-Scholes (BS) model and the actual call price also are presented in Figures 5.41, 5.43, 5.45, 5.47, 5.49, 5.51, and 5.53, after the plot of Model 5.5 only for GE options. Because the option samples are ordered by degree-of-moneyness first and by maturity month next, the systematic over- or underpricing of the Black-Scholes price across option positions can be observed easily from the plots. On the other hand, the prediction of Model 5.5 is more closely related to observed call value than that of the Black-Scholes price.

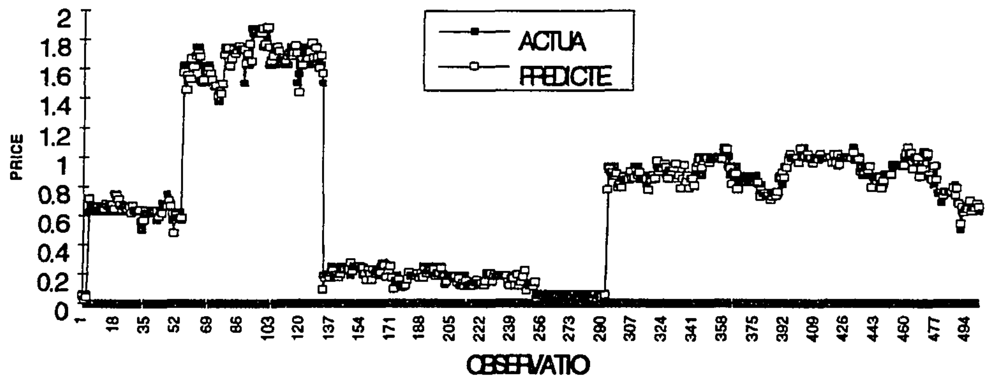


Figure 5.40 Actual and Model 5.5 Predicted Option Prices

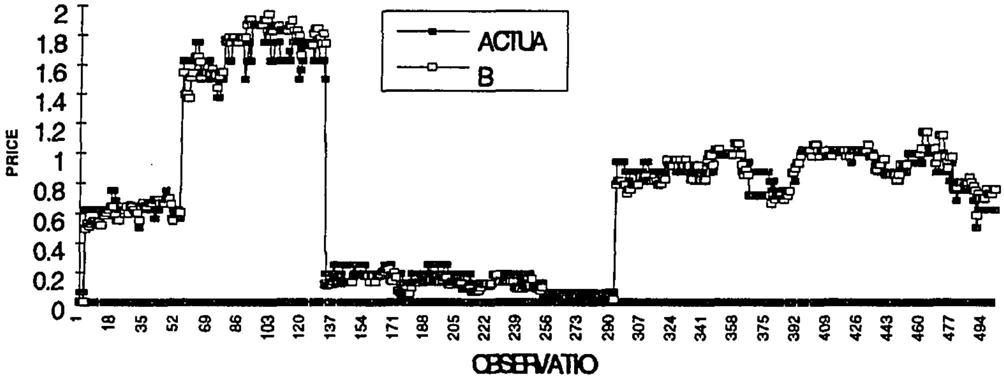


Figure 5.41 Actual and Black-Scholes Predicted Option Prices

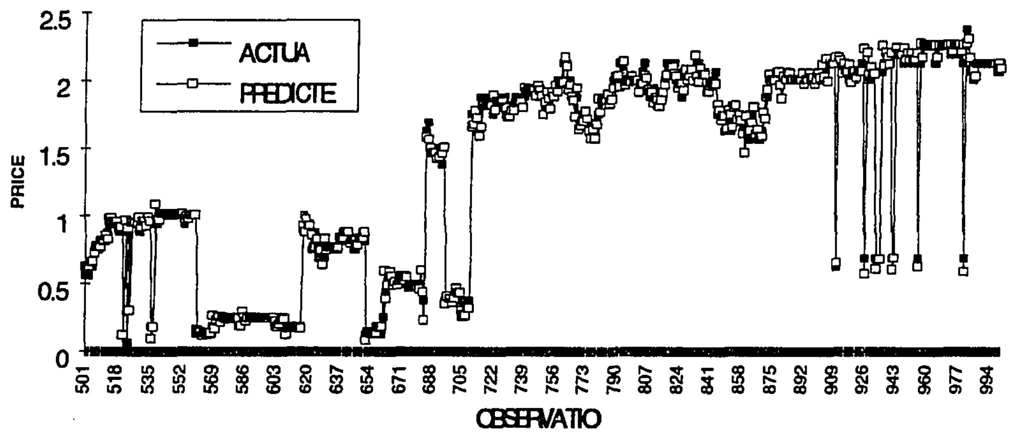


Figure 5.42 Actual and Model 5.5 Predicted Option Prices

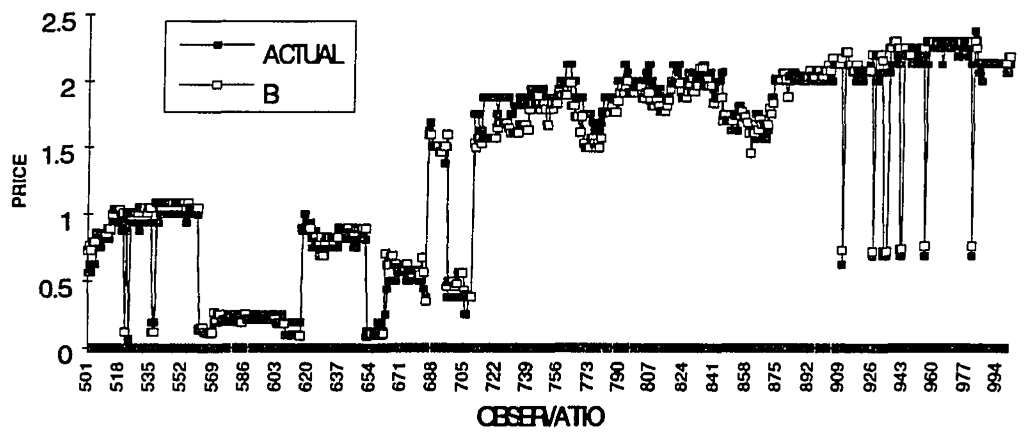


Figure 5.43 Actual and Black-Scholes Predicted Option Prices

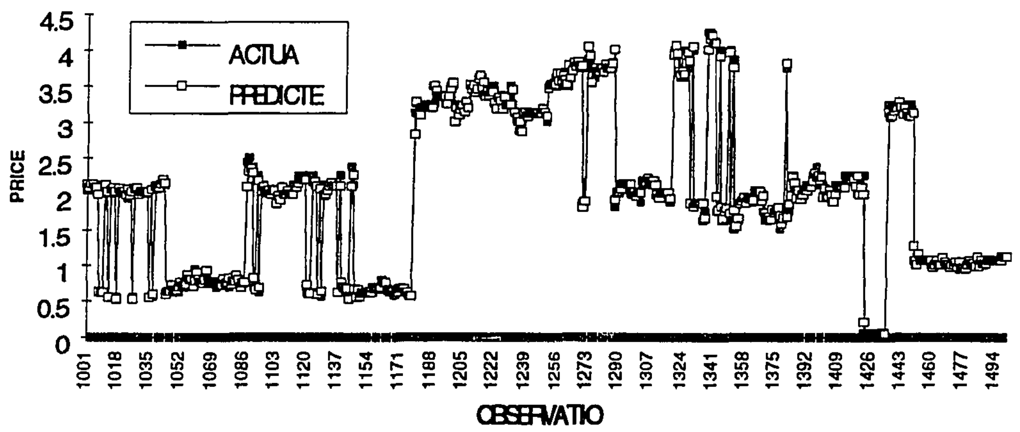


Figure 5.44 Actual and Model 5.5 Predicted Option Prices

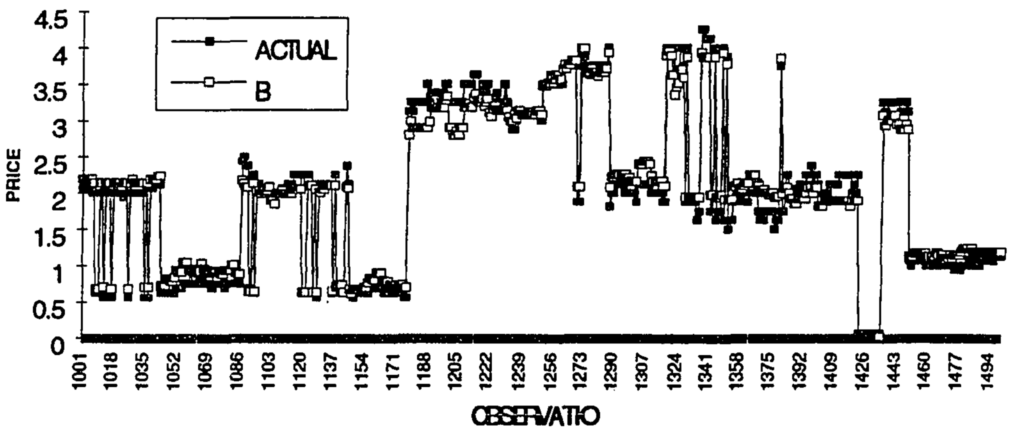


Figure 5.45 Actual and Black-Scholes Predicted Option Prices

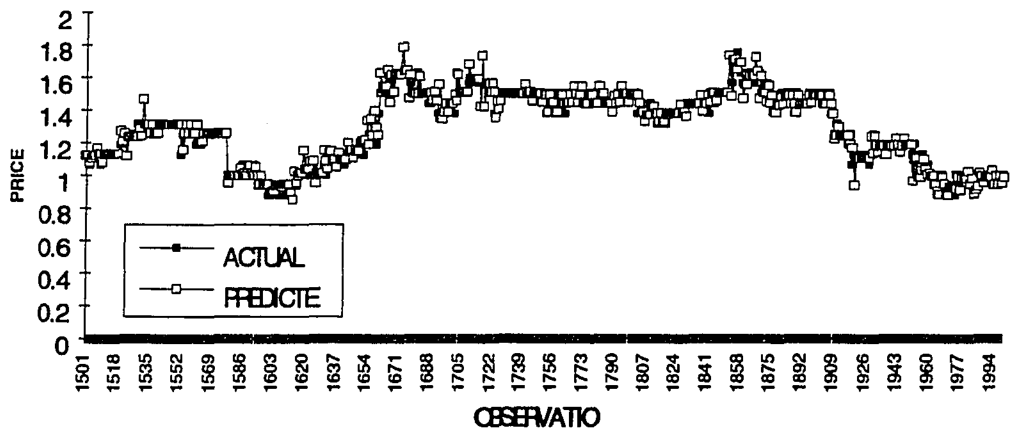


Figure 5.46 Actual and Model 5.5 Predicted Option Prices

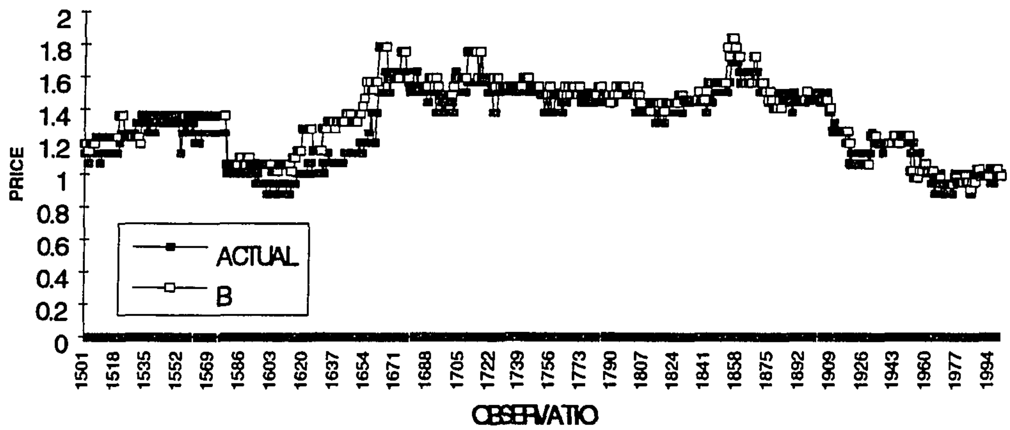


Figure 5.47 Actual and Model 5.5 Predicted Option Prices

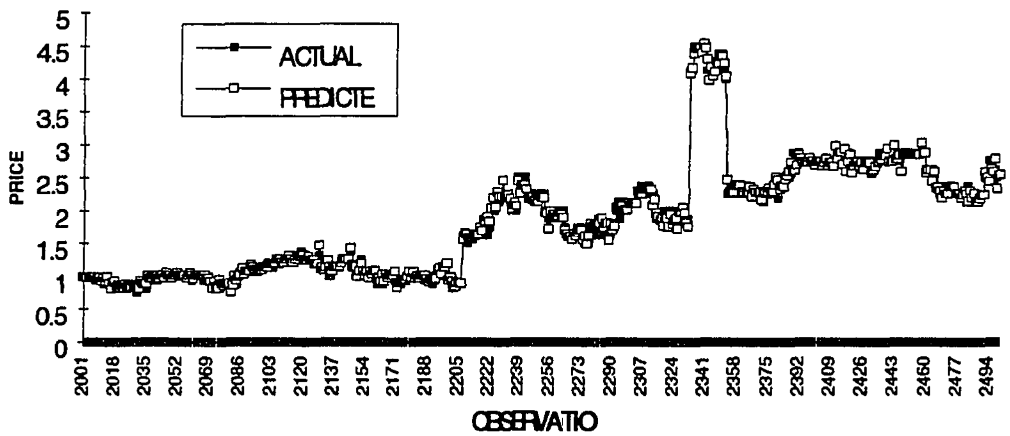


Figure 5.48 Actual and Black-Scholes Predicted Option Prices

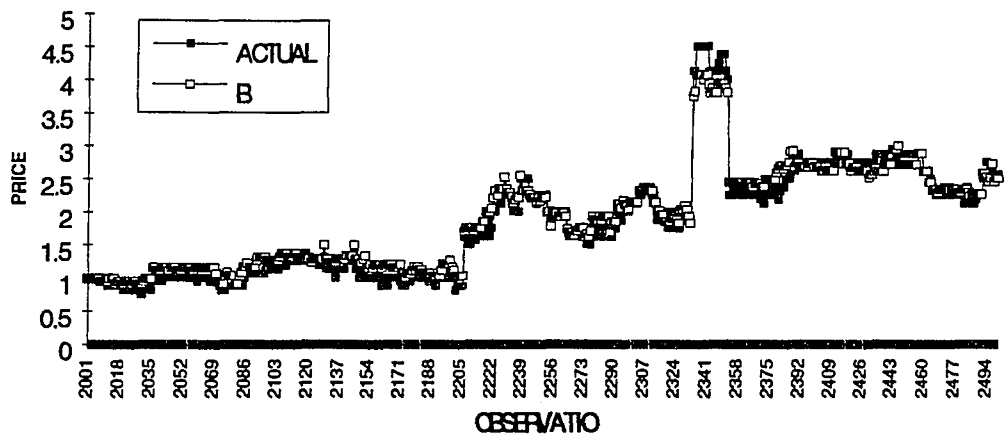


Figure 5.49 Actual and Black-Scholes Predicted Option Prices

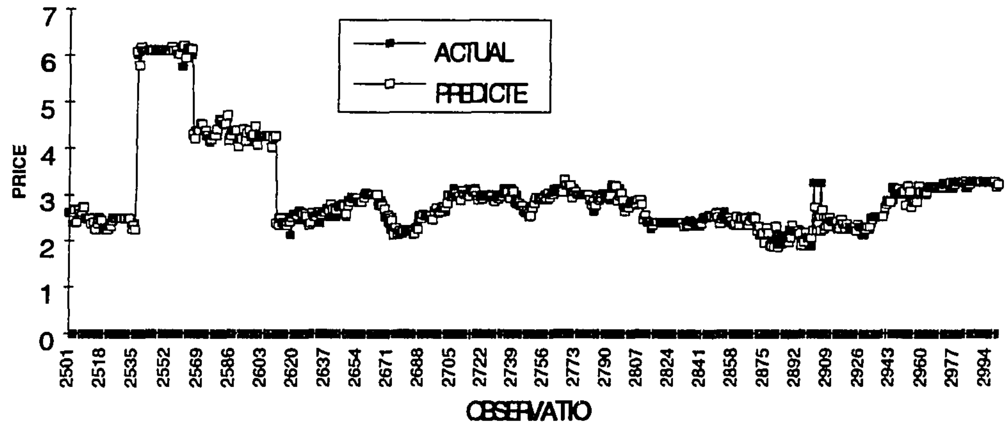


Figure 5.50 Actual and Model 5.5 Predicted Option Prices

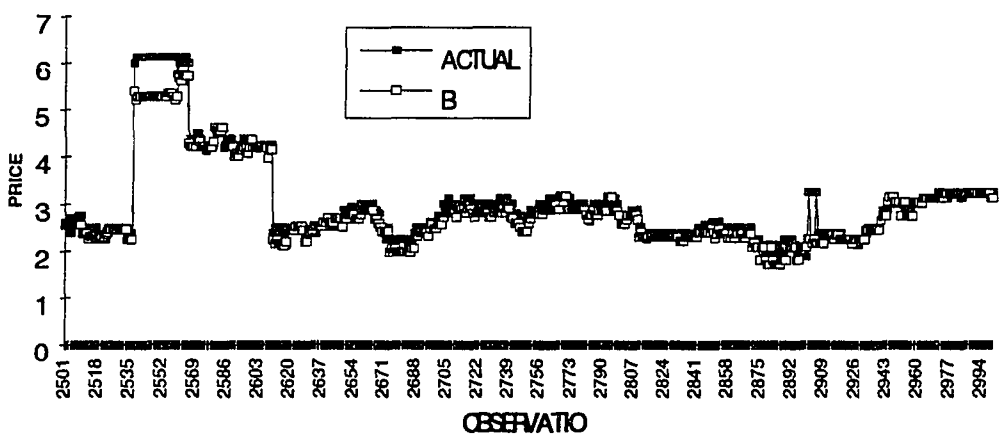


Figure 5.51 Actual and Black-Scholes Predicted Option Prices

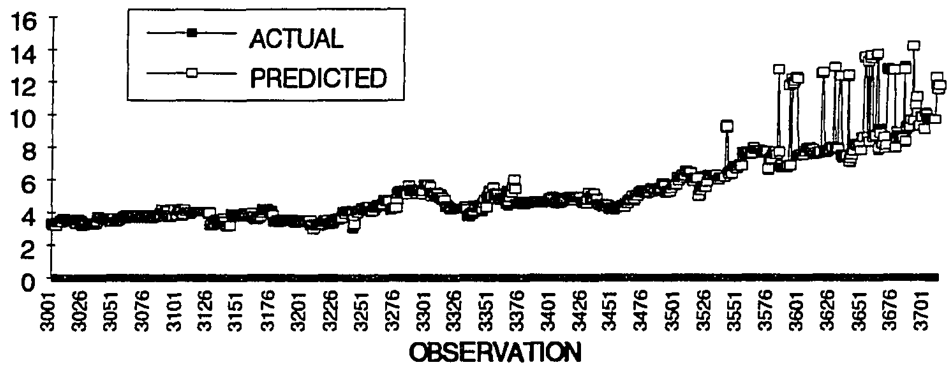


Figure 5.52 Actual and Model 5.5 Predicted Option Prices

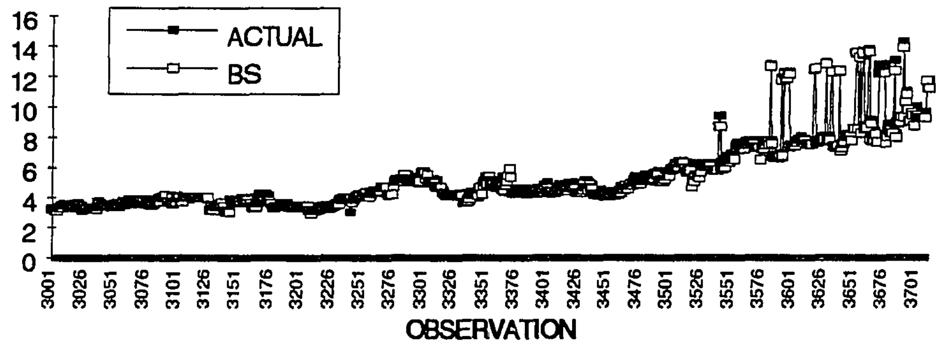


Figure 5.53 Actual and Black-Scholes Predicted Option Prices

Although various problems faced by the conventional analysis have been detected in the study by using Model 5.5, Section 5.4 shows that the Black-Scholes model price with flexible implied volatilities still exhibits pricing biases with respect to maturity and to exercise price. These biases, however, are inconsistent across underlying securities. But from the residual plots, the striking-price bias of the Black-Scholes model price with constant estimated implied volatility seems systematic for all stocks considered. These findings can be confirmed by computing the average mispricing of Model 9. In doing so, the results indicate strongly that, for all underlying securities, the out-of-the-money short maturity and also the in-the-money options are overpriced significantly. Thus, how the implied volatility is entered into the model has great impact. If only one estimated implied volatility is used to explain the option data, the model likely will exhibit a systematic striking-price bias for all underlying securities, as found by many researchers including Black (1975), MacBeth and Merville (1979), Gultekin et al. (1982), Sterk (1983), and Levy and Byun (1987). If various estimated implied voltilities are used, however, the striking-price bias may not be systematic across underlying securities, as found in this study and also in Whaley (1982). Nevertheless, the performance of Model 5.5 itself in predicting market option prices is quite impressive. The model eliminates totally any systematic mispricing that exists in the Black-Scholes model price at any maturity level or exercise price.

CHAPTER 6. PREDICTING OPTION PRICES

The robust model presented in the previous chapter is very useful for investigating the mispricing of the Black-Scholes model. The model expresses the market option price as a function of the Black-Scholes model price, striking price, and maturity. These last two arguments determine the direction of the mispricing which can be either positive or negative. If the mispricing is in the negative direction and also dominates the Black-Scholes model price, then the predicted price will be negative. Thus, an attempt to use the model of Chapter 5 to predict the price of a short maturity out-of-the-money option may produce a negative option price.

Another problem with the model of Chapter 5 is the discontinuity of the dummy variables used to describe the degree-of-moneyness and time-to-maturity. As the result of the discontinuity, the predicted option prices from the model will exhibit a jump from one category to another. In this chapter, we will construct a model for predicting option prices that maintains the basic structure of the robust model of Chapter 5 but does not have the two described undesirable properties.

Our objective is to construct a function that permits the option price to deviate from the Black-Scholes price, where the deviation is a smooth function of time-to-maturity, T_t, and degree-of-moneyness, m_t. Also, the predicted prices should always be nonnegative. A common procedure to guarantee a positive function, and the one we adopt, is to work with exponential functions.

We first define functions of m_t and T_t that we will use in our model. Let

$$m_{2t} = \begin{cases} m_t^2 & \text{for } |m_t| < 0.1 \\ 0.01 + 0.2[|m_t| - 0.1] & \text{for } |m_t| \geq 0.1 \end{cases} \tag{6.1}$$

and

$$T_{2t} = \begin{cases} 0 & \text{for } T_t > \overline{T}_3 \\ (T_t - \overline{T}_3)^2 & \text{for } \overline{T}_3 \leq T_t \leq \overline{T}_4 \\ (\overline{T}_4 - \overline{T}_3)^2 + 2(\overline{T}_4 - \overline{T}_3)(T_t - \overline{T}_4) & \text{for } T_t > \overline{T}_4 \end{cases} \tag{6.2}$$

where $\overline{T}_3$ and $\overline{T}_4$ are the average time-to-maturity of the option contracts that mature in maturity months $\tau = 3$ and 4, respectively.

The functions m_{2t} and T_{2t} are closely related to the quadratic function, they are continuous, and they have continuous derivatives. The values used to define m_{2t} and T_{2t} separate options into nine groups. With respect to striking price, options are separate into three groups; the out-of-the-money ($m_t \leq -0.1$), the at-the-money ($|m_t| < 0.1$), and the in-the-money ($m_t \geq 0.1$) options. With respect to time-to-maturity, options are also classified into three groups; the short ($T_t < \overline{T}_3$), medium ($\overline{T}_3 \leq T_t \leq \overline{T}_4$), and long maturity ($T_t > \overline{T}_4$). Notice that the function (6.1) is linear except when m_t is at-the-money. The function (6.2) is linear except for maturities in the medium time-to-maturity range. The three values of $\overline{T}_3$ are 0.154, 0.159, and 0.150 for GE, GM, and IBM, respectively. The three values of $\overline{T}_4$ are 0.253, 0.254, and 0.345 for GE, GM, and IBM, respectively.

Including the effects of striking price and time-to-maturity, our model for market option price is

$$\begin{aligned} C_t &= F(\sigma^2)e^{\alpha_1 m_{1t} + \alpha_2 m_{2t} + \beta_1 T_{1t} + \beta_2 T_{2t} + \gamma_1 m_{1t} T_{1t} + \gamma_2 m_{1t} T_{2t}} + \varepsilon_t , \\ \varepsilon_t &= \rho \varepsilon_{t-1} + \eta_t , \end{aligned} \tag{6.3}$$

where $F(\sigma^2)$ is the Black-Scholes function, $m_{1t} = m_t$, and $T_{1t} = T_t - \overline{T}_3$. This model contains the autoregressive representation for the errors introduced in Chapter 5. For this model, σ^2 can be considered to represent the implied volatility of the at-the-money short maturity options because m_{1t} , T_{1t} , m_{2t} and T_{2t} are all equal to zero for these options. The effect of a change in striking price, or of a change in time-to-maturity for each option class can be found by substituting the definition of the explanatory variables into Equation 6.3. For simplicity, consider only the exponent

$$g(m_t, T_t) \;=\; \alpha_1 m_{1t} + \alpha_2 m_{2t} + \beta_1 T_{1t} + \beta_2 T_{2t} + \gamma_1 m_{1t} T_{1t} + \gamma_2 m_{1t} T_{2t} \,. \qquad (6.4)$$

In the class of at-the-money short maturity options, $m_{2t} = m_t^2$, and $T_{2t} = 0$, and the exponent 6.4 is

$$\alpha_1 m_t + \alpha_2 m_t^2 + \beta_1 T_{1t} + \gamma_1 m_{1t} T_{1t} \,.$$

Thus, for these options, the percentage change in option prices due to the change in the extent to which option is in-the-money is measured by α_1 . The parameter, α_2 , measures the rate of change of option prices with respect to m_t . The percentage change in option prices due to the change in the time-to-maturity is measured by β_1 . The parameter, γ_1 , determines the interaction between m_t and T_{1t} in percentage terms. Table 6.1 contains the coefficients of m_t , m_t^2 , T_{1t} , T_{1t}^2 , $m_t T_{1t}$, and $m_t T_{1t}^2$ for the nine option classes.

6.1 Estimation and Prediction Results

It was decided to withhold a part of the data from the estimation to serve as a check on the predictive ability of the model. Since the data used in the study are limited, we removed only the last five observations in each option class for the prediction check. The model is estimated based on the rest of the data. The method for obtaining predictions is adapted from Fuller (1980).

Model 6.3 was fitted by the SYSNLIN procedure in SAS/ETS. The estimation results for GE, GM, and IBM options are reported in Table 6.2.

Table 6.1 The Effects of Striking Price and Time-to-Maturity

Option Class	m_t	m_t^2	T_{1t}	T_{1t}^2	$m_t T_{1t}$	$m_t T_{1t}^2$
At-the-Money Short Maturity	α_1	α_2	β_1	0	γ_1	0
In-the-Money Short Maturity	$\alpha_1 + 0.2\alpha_2$	0	β_1	0	γ_1	0
Out-of-the-Money Short Maturity	$\alpha_1 - 0.2\alpha_2$	0	β_1	0	γ_1	0
At-the-Money Medium Maturity	α_1	α_2	β_1	β_2	γ_1	γ_2
In-the-Money Medium Maturity	$\alpha_1 + 0.2\alpha_2$	0	β_1	β_2	γ_1	γ_2
Out-of-the-Money Medium Maturity	$\alpha_1 - 0.2\alpha_2$	0	β_1	β_2	γ_1	γ_2
At-the-Money Long Maturity	$\alpha_1 - \gamma_2(\overline{T}_4 - \overline{T}_3)^2$	α_2	$\beta_1 + 2\beta_2(\overline{T}_4 - \overline{T}_3)$	0	$\gamma_1 + \gamma_2 + 2(\overline{T}_4 - \overline{T}_3)$	0
In-the-Money Long Maturity	$\alpha_1 + .2\alpha_2 - \gamma_2(\overline{T}_4 - \overline{T}_3)^2$	0	$\beta_1 + 2\beta_2(\overline{T}_4 - \overline{T}_3)$	0	$\gamma_1 + \gamma_2 + 2(\overline{T}_4 - \overline{T}_3)$	0
Out-of-the-Money Long Maturity	$\alpha_1 - .2\alpha_2 - \gamma_2(\overline{T}_4 - \overline{T}_3)^2$	0	$\beta_1 + 2\beta_2(\overline{T}_4 - \overline{T}_3)$	0	$\gamma_1 + \gamma_2 + 2(\overline{T}_4 - \overline{T}_3)$	0

According to the statistics reported in Table 6.2, the estimated parameters that are significantly different from zero have the same sign in all three equations. To this extent, the results are consistent for the three securities.

The coefficient of T_{2t} , denoted by β_2 , is significant for none of the securities. However, γ_2 , the interaction of m_t with T_{1t}^2 , is significant in two of the three equations. Therefore, there is a significant nonlinear effect associated with time-to-maturity.

Recall that the variable T_{2t} is a grafted polynomial. It is quadratic for part of the medium maturity and linear for part of the short and long maturity. The coefficients of m_t , m_t^2 , T_{1t} , T_{1t}^2 , $m_t T_{1t}$, and $m_t T_{1t}^2$ for any degree-of-moneyness and maturity all can be found by substituting the values of the estimated parameters of Table 6.2 into the appropriate function in Table 6.1. The results are reported in Tables 6.3-6.5. The direction of the percentage changes (and the rate of the changes measured by the second power terms) in the market option price due to the changes in the extent to which the option is in-the-money and time-to-maturity (including the interactions) are almost identical for GE and IBM options. However, the direction of the percentage changes of the GM market prices is negative with respect to the increases in time-to-maturity, especially for long maturity options. In addition, the in-the-money long maturity GM options have a negative striking price effect. The effects of striking price (except for the in-the-money long maturity GM options), are similar for all option classes. Based on Model 6.3, neither the striking price nor maturity effect seems to be systematic across stocks. The results support the findings of the previous chapter that the bias patterns of the three securities are not the same (see also page 110).

Table 6.2 Summary Statistics for all Options

	GE		GM		IBM	
Parameter	Estimate	App. Std[a]	Estimate	App. Std[a]	Estimate	App. Std[a]
σ^2	.038*	.0003	.051*	.0005	.033*	.0001
α_1	.823*	.0596	1.153*	.0679	.595*	.0215
α_2	-4.253*	.3537	-6.57*	.4795	-3.366*	.1421
β_1	.084*	.0317	-.059	.0390	.044*	.0138
β_2	.391	.2240	-.306	.2710	-.065	.0623
γ_1	.369*	.1591	1.225*	.3123	.775*	.1415
γ_2	-5.171*	1.4311	-3.240	2.2737	-4.488*	.7390
ρ	.722*	.0115	.661*	.0131	.736*	.0043
MSE	.0089		.0082		.0147	
d.f.	3609		3309		25301	
R-square	.9976		.9971		.9978	
D.W.	2.327		2.291		2.530	

* The estimated parameter is significantly different from zero.
[a] The approximate standard deviation.

Table 6.3 The Effects of Striking Price and Time-to-Maturity for GE Options

Option Class	m_t	m_t^2	T_{1t}	T_{1t}^2	$m_t T_{1t}$	$m_t T_{1t}^2$
At-the-Money Short Maturity	.823 (.059)	-4.253 (.354)	.084 (.032)	0	.369 (.159)	0
In-the-Money Short Maturity	-.028 (.021)	0	.084 (.032)	0	.369 (.159)	0
Out-of-the-Money Short Maturity	1.674 (.129)	0	.084 (.032)	0	.369 (.159)	0
At-the-Money Medium Maturity	.823 (.0596)	-4.253 (.354)	.084 (.032)	.391 (.224)	.369 (.159)	-5.171 (1.431)
In-the-Money Medium Maturity	-.028 (.021)	0	.084 (.032)	.391 (.224)	.369 (.159)	-5.171 (1.431)
Out-of-the-Money Medium Maturity	1.674 (.129)	0	.084 (.032)	.391 (.224)	.369 (.159)	-5.171 (1.431)
At-the-Money Long Maturity	.874 (.060)	-4.253 (.354)	.161 (.022)	0	-4.604 (1.304)	0
In-the-Money Long Maturity	.023 (.032)	0	.161 (.022)	0	-4.604 (1.304)	0
Out-of-the-Money Long Maturity	1.724 (.128)	0	.161 (.022)	0	-4.604 (1.304)	0

The number in parenthesis is the approximate standard deviation

Table 6.4 The Effects of Striking Price and Time-to-Maturity for GM Options

Option class	m_t	m_t^2	T_{1t}	T_{1t}^2	$m_t T_{1t}$	$m_t T_{1t}^2$
At-the-Money Short Maturity	1.153 (.068)	-6.57 (.479)	-.059 (.039)	0	1.225 (.312)	0
In-the-Money Short Maturity	-.160 (.037)	0	-.059 (.039)	0	1.225 (.312)	0
Out-of-the-Money Short Maturity	2.467 (.162)	0	-.059 (.039)	0	1.225 (.312)	0
At-the-Money Medium Maturity	1.153 (.068)	-6.57 (.479)	-.059 (.039)	-.306 (.271)	1.225 (.312)	-3.240 (2.274)
In-the-Money Medium Maturity	-.160 (.037)	0	-.059 (.039)	-.306 (.271)	1.225 (.312)	-3.240 (2.274)
Out-of-the-Money Medium Maturity	2.467 (.162)	0	-.059 (.039)	-.306 (.271)	1.225 (.312)	-3.240 (2.274)
At-the-Money Long Maturity	1.183 (.068)	-6.57 (.479)	-.117 (.025)	0	-1.825 (1.994)	0
In-the-Money Long Maturity	-.131 (.054)	0	-.117 (.025)	0	-1.825 (1.994)	0
Out-of-the-Money Long Maturity	2.496 (.159)	0	-.117 (.025)	0	-1.825 (1.994)	0

The number in parenthesis is the approximate standard deviation

Table 6.5 The Effects of Striking Price and Time-to-Maturity for IBM Options

Option Class	m_t	m_t^2	T_{1t}	T_{1t}^2	$m_t T_{1t}$	$m_t T_{1t}^2$
At-the-Money Short Maturity	.595 (.022)	-3.366 (.142)	.044 (.014)	0	.775 (.142)	0
In-the-Money Short Maturity	-.078 (.012)	0	.044 (.014)	0	.775 (.142)	0
Out-of-the-Money Short Maturity	1.268 (.049)	0	.044 (.014)	0	.775 (.142)	0
At-the-Money Medium Maturity	.595 (.022)	-3.366 (.142)	.044 (.014)	-.065 (.062)	.775 (.142)	-4.488 (.739)
In-the-Money Medium Maturity	-.078 (.012)	0	.044 (.014)	-.065 (.062)	.775 (.142)	-4.488 (.739)
Out-of-the-Money Medium Maturity	1.268 (.049)	0	.044 (.014)	-.065 (.062)	.775 (.142)	-4.488 (.739)
At-the-Money Long Maturity	.765 (.038)	-3.366 (.142)	.019 (.013)	0	-3.323 (.612)	0
In-the-Money Long Maturity	.092 (.033)	0	.019 (.013)	0	-3.323 (.612)	0
Out-of-the-Money Long Maturity	1.438 (.057)	0	.019 (.013)	0	-3.323 (.612)	0

The number in parenthesis is the approximate standard deviation

The information in Tables 6.3-6.5 can be used to determine the changes of the option prices with respect to the changes in the degree-of-moneyness and time-to-maturity (T_{1t}) by computing the partial derivatives of the exponent of Equation 6.4 with respect to m_t and T_{1t}, respectively. The derivatives are computed at $(m, T) = (\alpha_1 + 2\alpha_2\, m, \beta_1 + \gamma_1\, m)$, $(\alpha_1 + 0.2\alpha_2 + \gamma_1 T, \beta_1 + \gamma_1 m)$, $(\alpha_1 - 0.2\alpha_2 + \gamma_1 T, \beta_1 + \gamma_1 m)$, $(\alpha_1 + 2\alpha_2 m + \gamma_1 T + \gamma_2 T^2, \beta_1 + 2\beta_2 T + \gamma_1 m + 2\gamma_2 mT)$, $(\alpha_1 + 0.2\alpha_2 + \gamma_1 T + \gamma_2 T^2, \beta_1 + 2\beta_2 T + \gamma_1 m + 2\gamma_2 mT)$, $(\alpha_1 - 0.2\alpha_2 + \gamma_1 T + \gamma_2 T^2, \beta_1 + 2\beta_2 T + \gamma_1 m + 2\gamma_2 mT)$, $\alpha_1 - \gamma_2 (\overline{T}_4 - \overline{T}_3)^2 + 2\alpha_2 m + (\gamma_1 + \gamma_2 + 2(\overline{T}_4 - \overline{T}_3))T, \beta_1 + 2\beta_2 (\overline{T}_4 - \overline{T}_3) + (\gamma_1 + \gamma_2 + 2(\overline{T}_4 - \overline{T}_3))m)$, $(\alpha_1 + 2\alpha_2 - \gamma_2 (\overline{T}_4 - \overline{T}_3)^2 + (\gamma_1 + \gamma_2 + 2(\overline{T}_4 - \overline{T}_3))T, \beta_1 + 2\beta_2 (\overline{T}_4 - \overline{T}_3) + (\gamma_1 + \gamma_2 + 2(\overline{T}_4 - \overline{T}_3))m)$, and $(\alpha_1 - 0.2\alpha_2 - \gamma_2 (\overline{T}_4 - \overline{T}_3)^2 + (\gamma_1 + \gamma_2 + 2(\overline{T}_4 - \overline{T}_3))T, \beta_1 + 2\beta_2 (\overline{T}_4 - \overline{T}_3) + (\gamma_1 + \gamma_2 + 2(\overline{T}_4 - \overline{T}_3))m)$ for the nine cells. The results are reported in Table 6.6. The table shows that the effect of m_t is positive for out-of-the-money options and the effect decreases as one moves to the in-the-money options. The effect of m_t is estimated to be negative for in-the-money options. This behavior is true for all the securities and all maturities. An increase in the time-to-maturity does not have a uniform effect on option prices. However, the derivative is negative for the in-the-money long maturity cell for all three securities.

Table 6.6 The Changes of Option Prices with Respect to m_t and T_{1t} (Derivatives of Equation 6.4)

Option Class	GE		GM		IBM	
	m_t	T_{1t}	m_t	T_{1t}	m_t	T_{1t}
At-the-Money Short Maturity	.729* (.064)	.086 (.070)	.729* (.077)	-.031 (.076)	.487* (.023)	.049* (.022)
In-the-Money Short Maturity	-.080 (.055)	.143 (.074)	-.309* (.090)	.145 (.082)	-.130* (.020)	.150* (.023)
Out-of-the-Money Short Maturity	1.620* (.156)	.043 (.142)	2.308* (.260)	-.263 (.239)	1.229* (.055)	-.048 (.041)
At-the-Money Medium Maturity	.899* (.063)	.131* (.015)	1.200* (.063)	-.099* (.012)	.713* (.026)	.032* (.007)
In-the-Money Medium Maturity	-.032 (.075)	.062* (.015)	-.086 (.116)	.028* (.005)	-.083* (.026)	-.106* (.016)
Out-of-the-Money Medium Maturity	1.674*[a] (.059)	.084*[a] (.031)	2.467*[a] (.067)	-.059[a] (.039)	1.268*[a] (.021)	.044*[a] (.013)
At-the-Money Long Maturity	-.076 (.080)	.212* (.079)	.980* (.093)	-.091 (.080)	-.246* (.030)	-.010 (.022)
In-the-Money Long Maturity	-1.019* (.065)	-.558* (.063)	-.544* (.103)	-.424* (.068)	-.800* (.029)	-.455* (.029)
Out-of-the-Money Long Maturity	1.150* (.129)	.640* (.052)	2.496*[a] (.067)	-.11*[a] (.039)	1.438*[a] (.021)	.019[a] (.013)

The number in parenthesis is the approximated standard deviation

[a] No observation in the cell ; * Significant at 5%

The Black-Scholes prices and the actual prices are computed to the predicted prices of Model 6.3 for the observations not used in the fitting. The Black-Scholes model prices are calculated based on the at-the-money implied volatility. The results are given in Tables 6.7-6.9 for GE, GM, and IBM options, respectively. The SSE given in the last part of the tables is the sum of squared deviations between the actual price and the Black-Scholes price and between the actual price and the price predicted by Model 6.3.

Table 6.7 Black-Scholes, Actual and Predicted Prices for GE Options

OBS	Black-Scholes Prices	Actual Prices	Predicted Prices	OBS	Black-Scholes Prices	Actual Prices	Predicted Prices
50	0.56831	0.625	0.46384	2331	1.95343	1.875	1.900425
51	0.62437	0.625	0.521014	2332	1.85038	1.875	1.794223
52	0.62437	0.625	0.524289	2532	2.48784	2.5	2.486327
53	0.62437	0.625	0.526652	2533	2.48784	2.5	2.476458
54	0.62437	0.5625	0.528358	2534	2.27911	2.375	2.247554
127	1.88527	1.75	1.760967	2535	2.27911	2.25	2.242413
128	1.81758	1.625	1.696604	2536	2.27911	2.375	2.238701
129	1.78435	1.625	1.666937	2606	4.23242	4.25	4.29404
130	1.85122	1.625	1.742327	2607	4.23242	4.25	4.325828
131	1.78391	1.5	1.673231	2608	4.01704	4	4.105459
287	0.00646	0.0625	0.036993	2609	4.19109	4.25	4.316943
288	0.00863	0.0625	0.030013	2610	4.19109	4.25	4.328896
289	0.01142	0.0625	0.026052	3123	4.01308	4	4.024291
290	0.00863	0.0625	0.019032	3124	4.01308	4	4.041825
291	0.01497	0.0625	0.021236	3125	4.01308	4	4.05448
657	0.1058	0.125	0.114295	3126	4.01308	3.875	4.063615
658	0.10264	0.1875	0.104088	3127	4.01308	4	4.070208
659	0.10264	0.1875	0.09851	3360	4.77621	4.875	4.951339
660	0.10264	0.1875	0.094485	3361	4.77621	5	4.935777
661	0.10998	0.125	0.098008	3362	4.77621	5	4.924544
705	0.44273	0.25	0.285993	3363	4.6823	5	4.818536
706	0.41155	0.25	0.283957	3364	4.5887	4.5	4.715098
707	0.39503	0.375	0.287703	3536	5.8581	6.25	6.206198
708	0.39503	0.375	0.301465	3537	5.8581	6.25	6.174583
709	0.39503	0.375	0.311398	3538	5.8581	6.25	6.151763
1173	0.75602	0.625	0.680284	3539	5.8581	6.25	6.135292
1174	0.75602	0.625	0.675075	3540	5.8581	6.125	6.123403
1175	0.75602	0.625	0.671315	3570	7.3114	7.75	7.577784
1176	0.71125	0.5625	0.625603	3571	7.3114	7.75	7.632518
1177	0.72349	0.5625	0.635917	3572	7.3114	7.75	7.672024
1378	1.99848	1.625	1.630868	3573	7.3114	7.75	7.70054
1379	1.99848	1.625	1.708551	3574	7.3114	7.75	7.721122
1380	3.90323	3.75	3.892033	3659	8.512	8.5	8.585025
1381	2.03465	1.75	1.843957	3660	8.712	8.75	8.80138
1382	2.10827	1.75	1.952468	3661	8.612	8.625	8.712896
1430	0.0032	0.0625	0.045829	3662	13.6109	13.75	13.656541
1431	0.00487	0.0625	0.035215	3663	13.6109	13.75	13.662598

Table 6.7 (continued)

OBS	Black-Scholes Prices	Actual Prices	Predicted Prices	OBS	Black-Scholes Prices	Actual Prices	Predicted Prices
1432	0.00728	0.0625	0.028607	3694	10.7737	11	11.053657
1433	0.02166	0.0625	0.03521	3695	9.8081	10	10.102248
1434	0.02166	0.0625	0.030685	3696	9.0267	9.25	9.324764
2202	1.0367	0.8125	0.930048	3697	9.0267	9.25	9.352756
2203	1.0367	0.875	0.947069	3698	9.611	9.75	9.974156
2204	1.0367	0.875	0.959355	3708	9.2684	9.625	9.673562
2205	1.0367	0.875	0.968222	3709	9.2684	9.625	9.708613
2206	1.0367	0.875	0.974623	3710	11.696	11.75	12.182343
2328	2.05982	2	2.00444	3711	11.2355	11.75	11.738593
2329	2.11426	2	2.064662	3712	11.2208	11.75	11.736393
2330	2.05982	1.875	2.009958	**SSE**	**4.119893**		**0.930724**

Table 6.8 Black-Scholes, Actual and Predicted Prices for GM Options

OBS	Black-Scholes Prices	Actual Prices	Predicted Prices	OBS	Black-Scholes Prices	Actual Prices	Predicted Prices
47	0.7729	0.5625	0.638808	1830	2.23826	2.375	2.27946
48	0.74752	0.625	0.623077	1831	2.23826	2.25	2.277574
49	0.76162	0.625	0.6442	1934	4.296	4.375	4.434228
50	0.76162	0.625	0.648179	1935	4.42221	4.625	4.565603
51	0.76162	0.625	0.650806	1936	3.66902	3.5	3.719215
105	1.59831	1.5	1.439143	1937	3.66902	3.5	3.714731
106	1.56233	1.8125	1.38081	1938	3.42288	3.25	3.43399
107	1.56233	1.4375	1.366854	2537	2.60981	2.5	2.5762
108	1.52688	1.25	1.321129	2538	2.60981	2.625	2.626528
109	1.56233	1.375	1.351548	2539	2.60981	2.625	2.659768
130	0.00582	0.0625	0.043925	2540	2.60981	2.5	2.681723
131	0.00398	0.0625	0.029174	2541	2.60981	2.75	2.696223
132	0.00854	0.0625	0.024977	2906	1.94304	2.0625	2.044965
133	0.00854	0.0625	0.019289	2907	1.87006	1.9375	1.955373
134	0.0011	0.0625	0.008365	2908	1.94304	2	2.025734
417	0.15481	0.25	0.186776	2909	1.94304	2	2.020682
418	0.1205	0.1875	0.1394	2910	1.94304	2.25	2.017345
419	0.1205	0.1875	0.129491	2940	2.71917	3.125	3.073704
420	0.1107	0.1875	0.113612	2941	2.83036	3.125	3.11321

Table 6.8 (continued)

OBS	Black-Scholes Prices	Actual Prices	Predicted Prices	OBS	Black-Scholes Prices	Actual Prices	Predicted Prices
421	0.07096	0.125	0.071854	2942	2.90243	3.375	3.135526
462	0.31643	0.4375	0.29022	2943	2.97532	3.5	3.177715
463	0.36824	0.5	0.335573	2944	3.02972	3.75	3.212733
464	0.36824	0.5	0.33276	3135	3.45477	3.5	3.703376
465	0.38687	0.5	0.348888	3136	3.36115	3.625	3.570845
466	0.33879	0.5	0.30224	3137	3.36115	3.625	3.550506
814	0.73471	0.75	0.717926	3138	3.36115	3.625	3.537073
815	0.64628	0.6875	0.610477	3139	3.29012	3.5	3.450206
816	0.64628	0.6875	0.596485	3143	4.65417	4.625	4.685046
817	0.62231	0.625	0.563731	3144	4.60608	4.625	4.671591
818	0.599	0.625	0.534851	3145	4.60608	4.625	4.693959
964	1.683	1.625	1.487484	3146	4.60608	4.625	4.708733
965	1.683	1.5625	1.479218	3147	4.67542	4.75	4.794897
966	1.64584	1.5625	1.43502	3291	7.50872	7.75	7.608372
967	1.82545	1.8125	1.621902	3292	7.60872	7.75	7.691537
968	1.82545	1.8125	1.61952	3293	7.50872	7.75	7.590136
1414	1.3678	1.375	1.386785	3294	7.50872	7.75	7.585345
1415	1.3678	1.3125	1.394569	3295	7.50872	7.75	7.582181
1416	1.3678	1.375	1.39971	3334	7.53134	7.5	7.601417
1417	1.3678	1.3125	1.403105	3335	7.52225	7.375	7.617415
1418	1.3678	1.375	1.405348	3336	6.70706	6.625	6.837447
1585	1.52543	1.625	1.604661	3337	6.29963	6.375	6.448378
1586	1.52543	1.625	1.591228	3338	5.90148	6.375	6.064678
1587	1.52543	1.625	1.582355	3386	7.3745	7.625	7.73288
1588	1.52543	1.625	1.576495	3387	7.3745	7.75	7.721572
1589	1.52543	1.875	1.572625	3388	7.3745	7.75	7.714104
1597	2.30201	2.75	2.620391	3389	7.27952	7.75	7.611914
1598	2.21824	2.75	2.444259	3390	7.3745	7.75	7.705913
1599	2.21824	2.625	2.38772	3418	7.05262	7.375	7.317628
1600	2.21824	2.5	2.350377	3419	7.02798	7.25	7.303879
1601	1.72066	2.0625	1.792569	3420	7.02798	7.25	7.311839
1827	2.41681	2.5	2.490086	3421	7.02798	7.25	7.317096
1828	2.47797	2.5	2.551047	3422	6.86989	7.5	7.155538
1829	2.23826	2.5	2.282317	**SSE**	**5.281217**		**2.204872**

Table 6.9 Black-Scholes, Actual and Predicted Prices for IBM Options

OBS	Black-Scholes Prices	Actual Prices	Predicted Prices	OBS	Black-Scholes Prices	Actual Prices	Predicted Prices
208	0.0084	0.0625	0.047643	19385	3.467	3.75	3.680821
209	0.0079	0.0625	0.036595	19386	3.467	3.75	3.629878
210	0.0079	0.0625	0.02882	19387	3.467	3.625	3.592362
211	0.0079	0.0625	0.023094	19388	3.467	3.75	3.564736
212	0.0088	0.0625	0.01978	19389	3.409	3.875	3.484333
237	0.2117	0.3125	0.255007	20048	5.7306	5.75	5.730835
238	0.2336	0.3125	0.259393	20049	5.6684	5.625	5.694421
239	0.2918	0.375	0.302778	20050	5.6065	5.5	5.650628
240	0.2918	0.375	0.293958	20051	5.6065	5.5	5.666594
241	0.2918	0.375	0.287463	20052	5.6065	5.5	5.678352
269	1.1286	1.1875	1.151469	20141	7.5777	7.625	7.558832
270	1.277	1.1875	1.271694	20142	7.5148	7.5	7.537117
271	1.2549	1.25	1.2303	20143	7.5148	7.625	7.569021
272	1.3219	1.3125	1.282383	20144	7.7412	7.75	7.827062
273	1.3679	1.3125	1.317631	20145	7.6142	7.75	7.713145
308	2.7073	2.75	2.716319	22318	3.6229	3.625	3.630286
309	2.7395	2.75	2.724726	22319	3.6229	3.5	3.634179
310	2.9044	2.875	2.863325	22320	3.6229	3.75	3.639157
311	2.9381	2.875	2.911696	22321	8.6218	8.625	8.699737
312	2.9044	2.875	2.85342	22322	3.6229	3.5	3.640712
1034	0.0141	0.0625	0.049653	24699	6.8396	6.875	6.802919
1035	0.0169	0.0625	0.042943	24700	6.8396	6.625	6.84189
1036	0.0169	0.0625	0.035976	24701	6.8396	6.125	6.870588
1037	0.0169	0.0625	0.030845	24702	6.8396	6.75	6.891721
1038	0.0141	0.0625	0.024316	24703	6.8396	6.75	6.907284
5326	0.7143	0.5	0.506248	24768	7.0699	8	7.588088
5327	0.7143	0.5	0.556875	24769	7.1532	8	7.581944
5328	0.7143	0.5	0.594157	24770	6.987	7.875	7.341563
5329	0.7143	0.5	0.621612	24771	6.7405	7.25	7.035581
5330	0.7143	0.5	0.64183	24772	6.3837	7.25	6.626749
6183	1.3582	1.3125	1.310273	25035	8.7033	8.75	8.875886
6184	1.3582	1.375	1.314011	25036	12.4162	13	12.71804
6185	1.3582	1.375	1.316764	25037	8.8592	9	9.088019
6186	1.3582	1.4375	1.318791	25038	8.8592	9	9.103444
6187	1.3582	1.375	1.320284	25039	8.9376	9.125	9.19653
7442	3.2796	3.125	3.158571	25094	10.0699	10.5	10.19452
7443	3.2796	3.25	3.183293	25095	10.5819	10.875	10.71819

Table 6.9 (continued)

OBS	Black-Scholes Prices	Actual Prices	Predicted Prices	OBS	Black-Scholes Prices	Actual Prices	Predicted Prices
7444	3.2359	3.125	3.155889	25096	10.7821	11.25	10.92631
7445	3.2359	3.125	3.169296	25097	10.7821	11.25	10.93368
7446	1.6981	1.625	1.592617	25098	10.7062	11.25	10.86475
7785	3.3134	3.125	3.157957	25155	12.5207	12.625	12.75325
7786	5.1694	5	5.13387	25156	13.0207	12.875	13.18548
7787	5.1197	5	5.099579	25157	13.5207	13.375	13.63132
7788	5.1197	5	5.11274	25158	13.5207	13.375	13.60126
7789	5.0211	4.875	5.018991	25159	13.6207	13.75	13.67634
12505	0.045	0.0625	0.073544	25351	11.7732	11.875	11.88333
12506	0.045	0.0625	0.065879	25352	16.5427	16.5	16.56446
12507	0.0365	0.0625	0.051736	25353	16.6427	16.75	16.66645
12508	0.0365	0.0625	0.047579	25354	11.6736	11.5	11.79844
12509	0.045	0.0625	0.053016	25355	16.6427	16.75	16.67222
18947	2.8572	2.5625	2.643016	25415	17.7833	17.75	18.11311
18948	2.8572	2.5625	2.702308	25416	12.8937	13.125	13.19438
18949	2.8572	2.625	2.745971	25417	12.6847	13	13.00682
18950	2.8572	2.625	2.778126	25418	12.6847	13	13.02384
18951	2.8572	2.625	2.801804	25419	12.6847	13	13.03637
				SSE	**7.31529**		**3.440291**

From Tables 6.7-6.9, on the average, the predicted prices of Model 6.3 are much closer to the actual option prices than are the Black-Scholes prices for all option classes and securities. The sum of squares of the prediction errors (SSE) of Model 6.3 is less than half of that for the Black-Scholes model. The model predicts GE options with an SSE that is one fourth that of the Black-Scholes model.

The predictions of Tables 6.7-6.9 used the full Model 6.3 which includes the previous price in the prediction through the Autoregressive model for the errors. The autocorrelation among the residuals of Model 6.3 plays an important part in reducing the prediction error of the model price. As can be seen from Table 6.10, the large difference in the prediction error sum of squares of Model 6.3 relative to the Black-Scholes model can be divided into two parts. If the prediction is constructed from Model 6.3 without the previous deviation, the SSE are those given in the second column. These SSE are 51% to 87% of those of the Black-Scholes model. The inclusion of the previous deviation in the prediction reduces the prediction SSE by about one half.

Model 6.3, which is based on the structure of the robust model from Chapter 5, out-performs the Black-Scholes model in predicting the future values of option prices. Although the robust model of Chapter 5 was useful in investigating the mispricing of the Black-Scholes model, the robust model may produce negative estimated option prices if it is used to predict very low values of market option prices. Model 6.3 has corrected this problem by placing the effects of maturity and striking price in an exponent. In addition, the effects of maturity and striking price of Model 6.3 are specified as the functions of continuous variables describing the behavior of time-to-maturity and degree-of-moneyness, respectively. Thus, the model has no jumps in the model prices between categories. The results show that the predictions of Model 6.3 are far better than those of the Black-Scholes model.

Table 6.10 The Sum of Squares of the Prediction Errors

Security	Prediction Error Sum of Squares		
	BS	Not Using Previous Deviation	One Period with AR
GE	4.12	2.09	.93
GM	5.28	3.96	2.2
IBM	7.32	6.37	3.44

CHAPTER 7. CONCLUSION

The discussion in Chapter 4 shows that the amount of work devoted to empirical testing of call options is rather impressive. While various approaches have been applied to different data sets and time periods, some common results appear to be dominating the studies.

First, with respect to model validity, the major conclusions are:

1. The Black-Scholes performs relatively well, especially for the at-the-money options. Deviations from model prices are consistently observed for deep-n and deep-out-of-the money options.
2. No alternative model consistently offers better predictions of market prices than the Black-Scholes model. There is some evidence to prefer the constant elasticity of variance model, but it is not conclusive.
3. The major problem faced by the Black-Scholes model, or any other model suggested so far, is the nonstationarity of the risk estimator of the underlying stock. The nature of the nonstationarity is not clear yet. Nevertheless, for short time periods, say of days, the Black-Scholes model may give good predictions of market prices and reveal under- and overvalued options.
4. Trading machanism and market synchronization may have affected the results of the tests. However, by using more detailed and accurate data, the major results were reconfirmed.
5. No one accounts for trasaction costs and taxes, which may affect the prices of traded options.

Second, the evidence so far is also consistent with rejecting the null hypothesis of market syschronization. The significant ex-post hedge returns reported by a few of the studies may indicate the lack of either trading synchronization or data synchronization. This conclusion is also reached from testing the boundary conditions.

Third, the results with respect to option market efficiency are not conclusive. While ex-ante hedge returns are usually found to be significant and hence indicate market inefficiency, they are completely eliminated after the proper adjustments for transaction costs for the most efficient trader are introduced. All agree that an outsider to the exchange cannot consistently make above-normal profits. A market maker who is affected by the bid-ask spread, and who incurs opportunity costs, may also find it difficult to generate above-normal net excess profits.

Our study examines pricing performance of the Black-Scholes model by constructing a robust model that corrects the problem of specification error faced by the conventional approach. The model includes the linear effect of striking price and maturity directly in to the option prices as well as allows option prices to be serially correlated. This approach not only improves the efficiency of the estimates but also reduces the bias of the regression variance and the coefficient estimates, particularly, the effect of striking price and maturity.

The development of the robust model starts with testing the implication of the Black-Scholes model assumption, i.e., the stability of implicit volatility over option positions. In order to eliminate the possibility that option variances may differ across maturity and/or exercise price, the weighted non-linear least squares is used to fit the three alternative models: (1) varying implied volatility over maturity month, (2) varying implied volatility over the in-the-money degree, (3) both (1) and (2) at the same time. The test results strongly indicate the inferiority of the Black-Scholes model in favor of the third alternative model. One of the reason that the third alternative model fits the data most may stem from the information content about dividend announcement that embeds in stock prices, and thus, the option prices. The other may arise from the distinct character of option prices themselves over their maturity and striking price. Thus, there is no one estimated implied volatility seem to be suitable in explaining the pricing behavior of the whole series of option prices.

Basing upon the acceptance of the Black-Scholes model with flexible implied volatility across maturity month and degree-of-moneyness, a robust model is built by incorporating the linear effect of exercise opportunity and time-to-expiration directly into the model. In addition, the error term of the model is assumed to follow the first order autoregressive (AR1) process. Without this modification, the Durbin-Watson test statistic is much smaller than two which indicates the problem of positive serial correlation. However, this problem vanishes after adding the autoregressive term.

The implication of the Black-Scholes assumption of constant volatility is retested basing upon the robust model and the model without the autoregressive error. The test results still lead to the rejection of the implication of the Black-Scholes assumption. Nevertheless, this rejection can not contribute to the claim that the variance rate of stock return follows the Ito' process.

Although, the robust model improves the efficiency of the estimates and also reduces the bias of the regression variance and the coefficient estimates, the Black-Scholes model price with flexible implied volatilities still exhibits pricing biases across the degree-of-moneyness and maturity. However, the biased patterns exhibiting in one underlying security differ from the other.

Most empirical studies use the at-the-money implied volatility as the estimated for the anticipated volatility of the stock rate of return, because the estimate seem to be stable over options' maturity. Thus, the study also investigates the stability of the estimated implied volatilities of the at-the-money options. After controlling for the effect of striking price and maturity (via robust model), the results show that within the class of the at-the-money itself (the at-the-money near in- or out-of-the-money), the estimated implied volatilities do differ across their degree of at-the-money and maturity (even for those whose time-to-maturity is at least ninety days to expiration).

In examining the ability of the two conventional approaches in identifying the mispricing of the Black-Scholes model, the results show that the regression approach may mislead the true mispricing behavior of the model price. Since, the simple regression technique can not classify options according to their degree-of-moneyness and maturity month, it can not identify the mispricing with respect to option positions. The average mean mispricing approach, however, may be able to indicate direction of mispricing of the model price across option positions. The precision of the estimates is much poorer than the results that can be obtained from the robust model.

In investigating the mispricing of the Black-Scholes model price using the conventional approach, holding the implied volatility fixed at the at-the-money level, the graphical examination reports that for the in-the-money (out-of-the-money) options, model estimates tend to be lower (higher) than market prices. Furthermore, the deep out-of-the-money options have tendency to be overpriced. These results are consistent for all securities considered. However, after adjusting for the model specification error, the mispricing of the (robust) model completely disappears. The systematic mispricing of the Black-Scholes model price induced by the conventional analysis with fixed estimated stock volatility may stem from functional form of the model price itself because `the Black/Scholes formula is non-linear in the variance, and unbiasedness is not preserved under a non-linear transformation' (Butler and Schachter 1986 p. 342).

The robust model developed in the study is not only useful for investigating the systematic mispricing of the Black-Scholes model but also improving the predictability of the call option price. Since, the model has detected the problem of specification error faced by the conventional, a reliable statistical inference can be drawn from the model. Furthermore, the residual sum of squares from the model reduces dramatically comparing to the one from the Black-Scholes model. Thus, a more precise estimate of option price can be obtained. However, as for any non-linear model, especially with time series, the estimation technique requires a large sample size which may not be able to obtain in such a short period study. Fortunately, option pricing theory has long developed the `put-call parity' which is a powerful tool in order to convert put price to be call price. Thus, the obstacle in estimating the robust model can be overcome by using this conversion.

Many studies recently used the at-the-money short maturity implied volatility to investigate the empirical impact of stock splits, stock dividends, and their announcements to the changes in the stock volatilities and option prices. Firms may decide to apply these policies in order to either keep the share prices within some `optimal' range, or convey favorable information to public. The ex-date effect of stock splits, stock dividends, and annoucements; as documented by Ohlson and Penman (1985), French and Dubofsky (1986), Sheikh (1989), and Chang and Chen (1989); would shift the rate of return variances of shares or the implied standard deviations upward. These results imply that production technology changes affect economic parameter, variance rate of returns. The increases of stock volatilities will increase the option returns. However, as shown in the study that the at-the-money short maturity implied volatilities may not be stable within its degree-of-moneyness. On the other hand, the robust model offers a way to estimate the implied volatility that is stable across striking prices and maturities. As the result, the estimated implied volatility obtained from the robust model may be more closely related to the variance rate of stock return than the at-the-money short maturity implied volatility. Thus, the implied volatility of the robust model may capture the impact of dividend policies far better than the implied volatility of the at-the-money short maturity options. Thus, the robust model is useful for further research in not only the option area but also in the stock area.

Although the robust model of Chapter 5 was useful in investigating the mispricing of the Black-Scholes model, the robust model may produce negative values of future option prices, especially, for short maturity or out-of-the-money options. However, the model can be modified to improve the prediction results by placing the effects of maturity and striking price in an exponent. In addition, the effects of maturity and striking price of the robust model are specified as the functions of continuous variables describing the behavior of time-to-maturity and degree-of-moneyness, respectively. Thus, the model has no jumps in the model prices between categories. The results show that the predictions of the modified robust model are more accurate than those of the Black-Scholes model.

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